SORT 47 (1) January-June 2023, 3-80 DOI: 10.57645/20.8080.02.1

Transport systems analysis: models and data

Jaume Barceló[1]

Abstract

Rapid advancements in new technologies, especially information and communication technologies (ICT), have significantly increased the number of sensors that capture data, namely those embedded in mobile devices. This wealth of data has garnered particular interest in analyzing transport systems, with some researchers arguing that the data alone are sufficient enough to render transport models unnecessary. However, this paper takes a contrary position and holds that models and data are not mutually exclusive but rather depend upon each other. Transport models are built upon established families of optimization and simulation approaches, and their development aligns with the scientific principles of operations research, which involves acquiring knowledge to derive modeling hypotheses. We provide an overview of these modeling principles and their application to transport systems, presenting numerous models that vary according to study objectives and corresponding modeling hypotheses. The data required for building, calibrating, and validating selected models are discussed, along with examples of using data analytics techniques to collect and handle the data supplied by ICT applications. The paper concludes with some comments on current and future trends.

MSC: *90B20, 90B10, 90-08, 90-C25, 90B06.*

Keywords: *Optimization, Simulation, Data Analytics, Traffic assignment, Traffic Simulation.*

1. Introductory remarks on system modeling and data

Operations research (OR) has been a scientific discipline since its inception, as noted by Blackett (1948), Ackoff, Gupta and Minas (1965), and it adheres to the methodological principles of science. Barceló (2015) states that "systems are observed; observations consist of measurements that are data or, in other words, facts from which laws can be derived and which can be articulated in the body of theories". An epistemological chain in

[1] Department of Statistics and Operations Research. Universitat Politècnica de Catalunya-UPC Barcelona Tech. Professor Emeritus.

Received: May 2023

which, for Mario Bunge (1960), the theories are usually formalized in terms of mathematical models, representing parts of the reality, which can be *descriptive* or *predictive*.

Barceló further elaborates that while a descriptive model explains what happens, a description is often insufficient and it is typically necessary to delve into how things happen and, if possible, why. This gives rise to a need for predictive models. In other words, science usually aims to be predictive. To these epistemological foundations, especially in the case of OR, we could also add: "the recognition of uncertainty as an inherent component of observed reality, and the falsifiability principle of Popper (1972), a cornerstone of the scientific method: the theories or models are true until further empirical evidence proves them false".

In accordance with Heavens, Ward and Natalie (2013), we adopt the perspective that "a model formally organizes what we know, or we think we know, about a system to predict how it might behave in the present, future or past, as well as how it might respond to external influence" Furthermore, the first methodological step in studying a system involves data acquisition, which is carried out in accordance with the study objectives and available technologies for making observations, specifically through measurements. The underlying assumption of this step is that data contain information about the phenomenon under study, for which the data must be suitably processed and analyzed in order to find the necessary information. In other words, we can deduce how systems work by acquiring knowledge about them and translating this understanding into "laws" or modeling hypotheses, with the formal structure of these hypotheses defining the system model. In what follows, our present research on transport systems will employ mathematical formalism in terms of equations while also relying on implicit representations such as simulation models.

The model-building process consists of translating the modeling hypothesis, derived from the acquired knowledge, into appropriate formal terms that align with the study objectives. In the realm of operations research or similar perspectives adopted when analyzing transport systems, the models of analyzed systems serve a crucial purpose. Beyond acquiring knowledge about the studied system, their primary objective is to generate new insights by answering what-if questions regarding the system's response to external influences, such as transport policies that can impact its behavior. In other words, in the context addressed in this paper, a key objective of the modeling task is to utilize the model as the core component of a decision support system, aimed at facilitating optimal decision-making to enhance the system's response. Figure 1 provides a conceptual diagram for visualizing this methodological process.

Data have traditionally been scarce and costly, especially in the case of transport systems. However, this situation has dramatically changed with the advent and widespread adoption of new information and communication technologies (ICT), such that we are often overwhelmed by the deluge of data. This has led some to claim that the scientific method is no longer necessary. As Anderson (2008) famously argued, "Petabytes allow us to say: 'Correlation is enough.' [...] Correlation supersedes causation, and science can advance even without coherent models, unified theories, or really any mechanistic

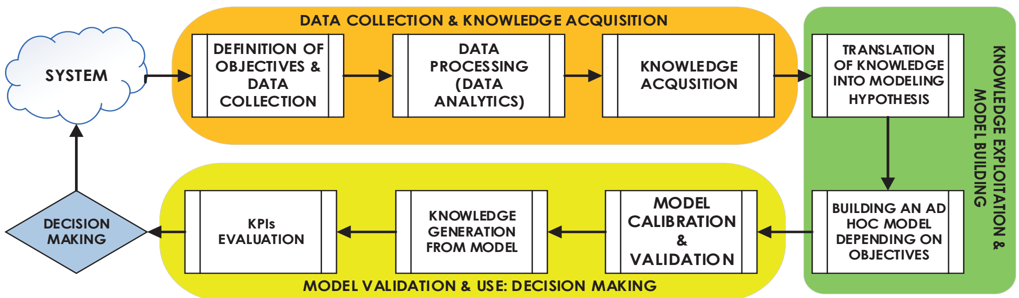

Figure 1. *Our methodological scheme for building and using models.*

explanation at all." Nevertheless, it is essential to heed the advice of the International Transport Forum (ITF, 2015), among many others. In the specific context of transport systems, ITF warns that the availability of massive and near real-time datasets can create the illusion of mirroring reality and tempt us to assume that data alone provide an accurate representation of reality. Consequently, there is a perceived notion that we can dispense with classic statistical tests regarding bias, validity, explanatory theories, and models. However, numerous examples demonstrate that data analytics have failed to provide long-term robust predictive results. Therefore, there is a need for algorithms that consistently detect patterns and mitigate bias in the data. These techniques are well suited to discovering less obvious or even hidden correlations in the initial data. We must bear in mind not only that correlation does not imply causation, but that they are completely different. Although correlated variables may reveal a possible causal relationship in the data, they do not explain which correlations are meaningful or predictive. Even if a correlation proves to be robust over a given period, data analytics alone cannot provide insights into factors that may cause the correlation to break down or lead to the emergence of new patterns.

The approach adopted in this paper assumes that both data and models are necessary. This notion is succinctly encapsulated by an anonymous quotation, which states that "Data without models are just numbers, but models without data are just stories."

The methodological approach summarized in Figure 1 highlights another relevant aspect: the objectives driving the analysis of the system.

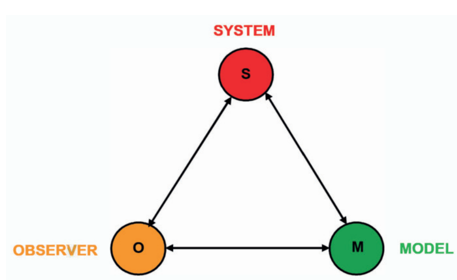

Figure 2. *The Minsky triad.*

These objectives serve as a guiding force for determining the relevant and necessary data to collect, as well as for constructing the system model. Thus, different models of the same system may exist, depending on the study objectives. Minsky's proposal in 1965 (Minsky, 1965) has come to be known as the Minsky triad of system, observer, and model (Figure 2), which we can summarize by saying that an object M is considered a model of a system S if it can provide valid answers to the questions posed by an observer O of that system.

Consequence: There is no such a thing as the unique model of a system.

This paper is structured as follows. Section 2 demonstrates how to apply the Minsky triad to transport systems, presenting a summary of the main alternative modeling approaches used in transport systems analysis. These approaches vary depending on the objectives and underlying modeling hypotheses. The section identifies the data requirements for each type of model and discusses the role played by data and the criteria for determining what makes a model useful. Section 3 provides some examples of the data needed to build these models, as well as how they are sourced through ICT applications and the necessary data analytics processes that render them usable. Section 4 summarizes the main conclusions and recommendations drawn from the paper, along with an overview of current and future trends in transport modeling.

2. Modeling transport systems

The pervasive penetration of the automobile as a private motorized transportation mode was driven by the economic interests of major manufacturers in the 1930s and gained momentum following the Second World War, resulting in a profound social and urban transformation of cities and metropolitan areas. The phenomena of urban sprawl described by Barceló (2019) emerged as a consequence of an unplanned and anarchic expansion due to a combination of factors: the relative affluence shifting from rural to city populations, lifestyle changes, and, particularly, advancements in individual motorized mobility. The latter factor led to a spatial separation of residential and working areas, made possible by the development of transportation systems that in turn led to the well-known consequences that we call traffic congestion.

The main consequence was a substantial demand for the expansion of road networks, thereby necessitating the development of appropriate tools for rational planning processes to assist decision-makers in determining which infrastructures to develop and how to meet the growing demand. Almost at the same time, the escalating traffic congestion sparked interest in understanding the dynamics of traffic flows and causes of congestion. The hope was that better comprehension of these factors would improve management policies and possibly alleviate the negative impacts associated with congestion.

Depending on the objectives, various models have been developed to analyze transport systems. This section provides a concise overview of the primary models that reflect a complementary understanding of transport systems, their associated modeling hypotheses, and their translation into mathematical models. Additionally, it provides insight into the data required to support these models, which are the following:

a. Models aimed at understanding how travelers use the existing road transport network to navigate from their origins to destinations in a given geographic area. These models support the long-term planning of transport infrastructures and are known as "static traffic assignment models."

b. Models for understanding the dynamics of traffic flows.

- b.1 Macroscopic models based on traffic flow theory, with an aggregated perspective of flow dynamics.
- b.2 Microscopic models that describe flow dynamics by considering the individual components constituting the flow.

c. Models explicitly accounting for the dynamics of traffic flows.

- c.1 Dynamic assignment models that explicitly account for time dependencies. These either analytically describe traffic flows or approximate their dynamics through simulation.
- c.2 Microscopic simulation models that capture the individual dynamics of vehicles within the traffic flow.

Note: The following focuses solely on the traffic flows of passenger cars. Public transport requires a similar but distinct modeling approach with specific features that are different from passenger cars. Including models for public transport in this paper would make it excessively long.

2.1. Static traffic assignment models

Transportation analysis typically revolves around understanding traffic patterns in a given geographic area, most frequently an urban or metropolitan area spanned by a transportation network. The goal is to gain insight into how the transport demand (i.e., the volume of trips in an area) uses the transport infrastructure under certain conditions. This transport demand is commonly defined in terms of an origin-to-destination (OD) matrix, X, whose entries (r,s) represent the number of trips from an origin r to a destination s. From a practical point of view, the study area is split into many transport analysis zones (TAZ) using well-established criteria that consider factors such as surface area and socioeconomic data obtained from various sources, for example, census tracts or population statistics (Ortúzar and Willumsen, 2011).

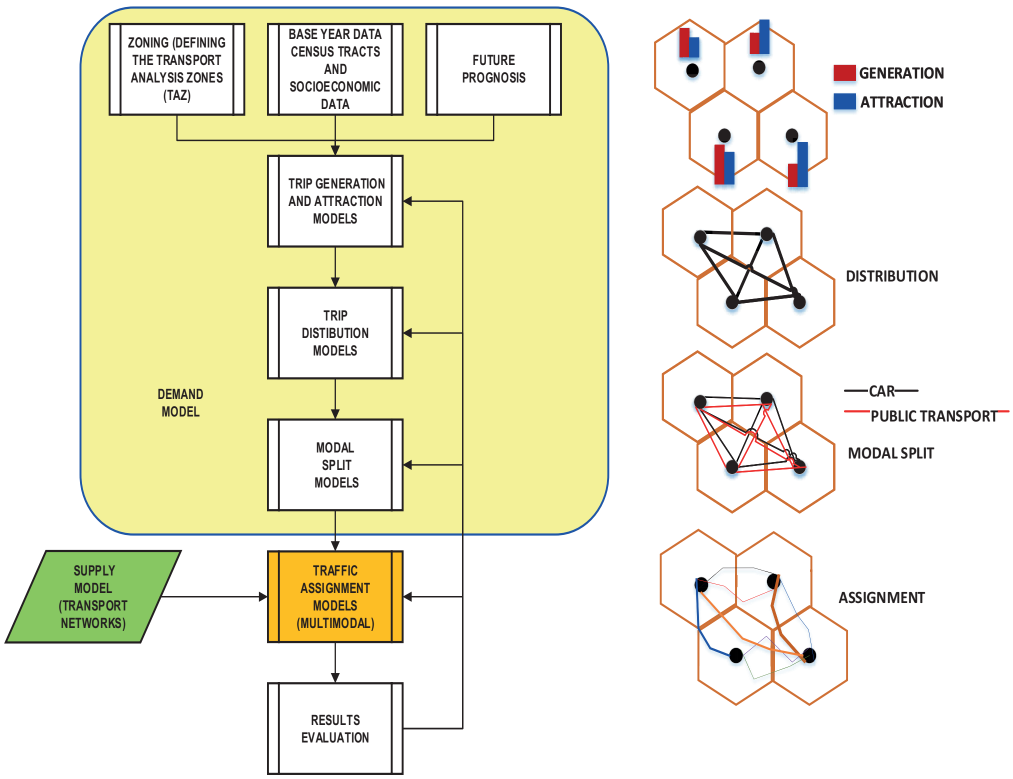

Figure 3. *Conceptual diagram of the four-step model.*

Figure 3 conceptually summarizes the logic diagram of the conventional planning process approach known as the *four-step model*, which has become a standard in transport modeling since the Second World War, due to a need to manage the consequences of post-war development and economic growth. It is based on the landmark study by Mitchell and Rapkin (1954), who applied analytical methods to establish a comprehensive framework. However, this model has deficiencies and limitations that have been widely discussed (McNally, 2000). The reasons for mentioning this model are manifold. First, it provides an overview of the modeling exercise described in Section 1, applied specifically to transport systems. Second, it helps identify the comprehensive path of intermediate models that led to the targeted model in this paper, which is the traffic assignment model. Finally, it serves to highlight the data requirements and the type of data needed to render the model operational.

In summary, assuming a homogeneous zonal splitting of the TAZ and using associated socioeconomic attributes such as population and economic activities, econometric models can be constructed to estimate the total number of trips O_r generated by a given origin r within a specific period (e.g., an average working day), considering all possible destinations within the other TAZ of that area. Similarly, equivalent attributes allow building econometric models to estimate the total number of trips D_s attracted by a given destination s. Building such generation and attraction models for all origins and destinations is the first step. The second step consists of estimating the number of trips x_{rs} leaving origin r for destination s, which yields an estimation of the origin to destination (OD) matrix M of the total number of trips between all OD pairs (r,s) without distinguishing which transportation mode is being used. The third step involves the *modal split model*, whose purpose is to discriminate among available transportation modes, such as passenger cars, public transport bus, or public transport metro. The modal split model typically relies on discrete choice models that consider the perceived utilities of the travelers, as discussed by Ben-Akiva and Lerman (1985), Ben-Akiva and Bierlaire (1999). In the case of passenger cars, the fourth step is the traffic assignment step.

Traffic assignment refers to the process of allocating traffic demand, represented by an origin-destination matrix, onto the transportation network. This enables computing traffic flows on network links and offers insights into trip behavior and accessibility to activity locations. Figure 4 illustrates the four steps applied to the *first crown* of the Metropolitan Area of Barcelona, where first crown refers to the continuum of the 18 most populated municipalities including the city.

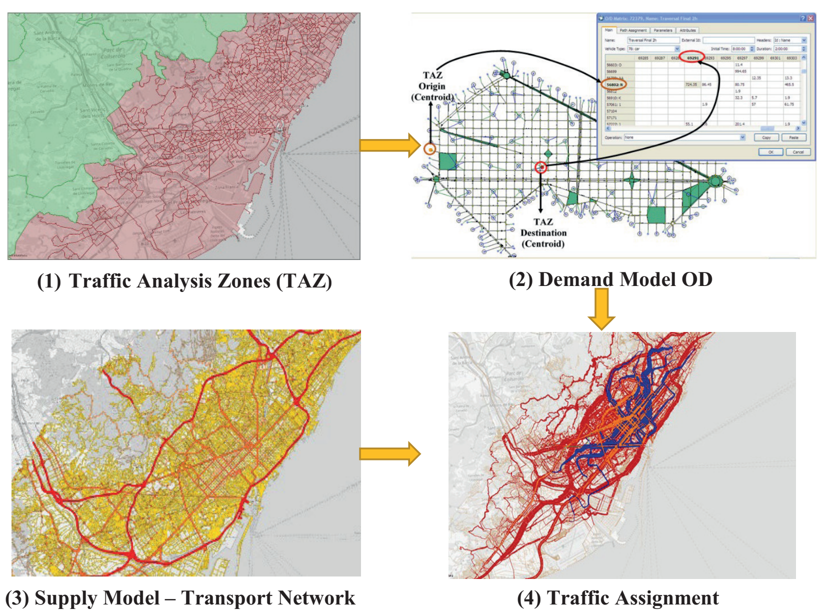

Figure 4. *(1) The partitioning of the first crown into TAZ. (2) The correspondence between the centroids representing an origin-destination pair for the subarea corresponding to the central business district of the city and the associated OD matrix. (3) Highlighting of the main arterials of the road network of the first crown. (4) The traffic flows resulting from a multimodal traffic assignment, with car traffic flows in red, bus flows in blue, and metro flows in orange. The thickness of the lines scales the intensity of the depicted flows.*

The underlying modeling hypothesis is that travelers move from origins to destinations in the network by selecting available routes based on behavioral choices governed by certain rules. The characteristics of a traffic assignment procedure are determined by this key modeling hypothesis that is based on the concept of user equilibrium, which assumes that travelers try to minimize their individual travel times by choosing what they perceive to be the shortest routes under prevailing traffic conditions. This modeling hypothesis is formulated in terms of Wardrop's first principle (Wardrop, 1952):

> The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route.

Traffic assignment models based on this principle are known as user equilibrium models, which differ from models whose objective is to optimize the total system travel time independently of individual preferences. For a more in-depth exploration that will not be considered here, see Sheffi (1985), Florian and Hearn (1995), and Patriksson (1994). Florian and Hearn (1995) demonstrated that when the path flows x_{rsp} from origin r to destination s along path p, with path costs tt_{rsp}, they must satisfy:

$$(tt_{rsp} - \theta_{rs})\ x_{rsp} = 0\ \ \forall p \in K_{rs}\ \forall (r,s) \in I \tag{1}$$

$$tt_{rsp} - \theta_{rs} \geq 0\ \ \forall p \in K_{rs}\ \forall (r,s) \in I \tag{2}$$

$$tt_{rsp},\ \theta_{rs},\ x_{rsp}\ \geq 0\ \ \ \forall p \in K_{rs}\ \forall (r,s) \in I \tag{3}$$

and the flow balancing equations:

$$\sum_{\forall p \in K_{rs}} x_{rsp} = X_{rs},\ \forall (r,s) \in I \tag{4}$$

where θ_{rs} image1 is the cost of the shortest path from r to s, K_{rs} is the set of all available paths from r to s, I is the set of all origin-destination pairs (r,s) in the network and X_{rs} is the demand (number of trips) from r to s. Then, these flows are in an equilibrium that satisfies Wardrop's principle. Effectively, if path p from origin r to destination s carries a flow $x_{rsp} > 0$, then the first equation is satisfied only if the path cost tt_{rsp} is equal to the minimum path cost θ_{rs} , that is, $tt_{rsp} - \theta_{rs} = 0$ for all paths from r to s, as required by Wardrop's principle. Reciprocally, if the path cost tt_{rsp} is greater than the minimum path cost θ_{rs}, that is $tt_{rsp} - \theta_{rs} > 0$, then satisfying the first equation requires that the flow on path p from r to s be zero, which is in other words an unused path according to Wardrop's principle.

Constraints (4) determine when a flow is feasible or not in terms of flow balance. If K_{rs} is the set of all paths for the (r,s) OD pair, then the sum of flows on paths for the (r,s) OD pair must be equal to the demand X_{rs}, which is the total number of trips for that OD pair. Applying some algebra (Florian and Hearn, 1995; Patriksson, 1994), the static traffic assignment model can be formulated in the space of the path feasible flows $\aleph$ in terms of the following system of variational inequalities:

$$T(X^*)(X - X^*) \geq 0,\ \forall X \in \aleph \tag{5}$$

where $T(X^*)$ is the vector of path flows, X^* is the optimal path flow, and $\aleph$ is the set of feasible path flows.

$$\aleph = \left\{ x_{rsp} \middle| \sum_{\forall p \in K_{rs}} x_{rsp} = X_{rs},\ \forall (r,s) \in I\ \ ,\ x_{rsp} > 0 \right\} \tag{6}$$

We assume that the road network is modeled in terms of a graph $G = (N, A)$ with a set of nodes N representing either intersections or dummy nodes associated with the transportation zones (usually referred to as centroids), and a set A of arcs used to model the infrastructure and the connections between centroids to the network. Thus, considering the relationships between path flows x_{rsp} and link flows v_a, $\forall a \in A$, we have:

$$v_a = \sum_{(r,s) \in I} \sum_{p \in K_{rs}} x_{rsp} \delta_{ap} \quad where \begin{cases} 1 & \text{if arc a belongs to path } p \text{ from } r \text{ to } s \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where δ_{ap} are the entries of the link-path incidence matrix. Assuming that the relationship between path costs tt_{rsp} and link costs $c_a(v)$, then:

$$tt_{rsp} = \sum_{a \in A} \delta_{ap} c_a(v) \tag{8}$$

where the link cost of each arc a $c_a(v)$, $\forall a \in A$ is a function of the vector of feasible flows v in all arcs. Again, after some algebra (Florian and Hearn, 1995; Patriksson, 1994), the equivalent formulation of the model (5) in the space of link flows is:

$$C(v^*)\left[v - v^*\right] \geq 0,\ v \in V \tag{9}$$

$$V = \left\{ v : v_a = \sum_{(r,s) \in I} \sum_{p \in K_{rs}} x_{rsp} \delta_{ap}\ ,\ \sum_{\forall p \in K_{rs}} x_{rsp} = X_{rs},\ x_{rsp} \geq 0,\ \forall (r,s) \in I \right\} \tag{10}$$

where V is the set of feasible flows. This the Smith's (1979) variational inequality. It can be proven that there is no equivalent convex optimization problem unless the cost functions $c_a(v)$ are separable, meaning their Jacobian is symmetric (Florian and Hearn, 1995). The simpler separability condition holds when they depend only on the flow in the link:

$$c_a(v) = c_a(v_a),\ \forall a \in A \tag{11}$$

and demands X_{rs} are considered constant, independent of travel costs. Thus, the variational inequality formulation has the following equivalent convex optimization problem (Patriksson, 1994; Florian and Hearn 1995):

$$\begin{aligned} &\text{Min } C(v) = \sum_{a \in A} \int_0^{v_a} c_a(x) dx \\ &s.t. \sum_{\forall p \in K_{rs}} x_{rsp} = X_{rs},\ \forall (r,s) \in I \\ &(x_{rsp} \geq 0\ \forall (r,s) \in I,\ p \in K_{rs} \end{aligned} \tag{12}$$

and the definitional constraint of v_a (7).

Assuming that the separability conditions hold and that therefore the link cost functions of each link a depend only on the volume v_a in that link, a critical aspect of the model-building process is determining the specific form of the functions $c_a(v_a)$. This assumption is also relevant for numerically solving the model's algorithms. The link cost functions $(c_a(v_a)$, also known as *volume delay functions* (VDF), quantify the variation in travel time within a link a according to the traffic volume v_a. This reflects the dependencies between a link's travel time and its traffic flow, although the link cost functions can also include additional factors indicating the cost (or impedance) of using a link, such as toll fares in urban pricing systems. In such cases, the functions represent the generalized costs of using the link, and they can be interpreted as indicators of the level of service provided by the link. Empirical studies suggest that these costs play a crucial role when users decide which routes to take.

The first form proposed for the VDF functions was that introduced by the Bureau of Public Roads in 1964 and has since become widely known as BPR functions, which are still widely used in the current transportation modeling practice. They take the analytical form:

$$c_a(v_a) = t_0^a \left[1 + \alpha_a \left(\frac{v_a}{\kappa_a} \right)^{\beta_a} \right] \tag{13}$$

where t_0^a represents the minimum time to traverse link a, which traffic engineers call the free flow time, which represents the time it takes for a to travel freely through the link without competing with other vehicles for the available capacity. κ_a represent the capacity or maximum flow of link a; and α_a and β_a are link-specific parameters that must be calibrated.

Although BPR functions are appealing for their simplicity and relative ease of calibration, they nevertheless exhibit anomalous behavior in certain circumstances. For example, when the β_a is large, it provides abnormal values for those links with $\frac{v_a}{\kappa_a} > 1$ in the first iterations of the assignment algorithms. This delays the convergence due to the anomalous overload of those links. Additionally, for links with traffic loads well below their capacity, the link travel time remains nearly equivalent to the free flow time regardless of the actual flow. These shortcomings prompted the search for more sophisticated functions that overcome these drawbacks. One well-known family of functions, widely used in practice, is the family of conical functions proposed by Spiess (1990):

$$c_a(v_a) = t_0^a \left[2 - \beta_a - \alpha_a \left(1 - \frac{v_a}{\kappa_a} \right) + \sqrt{\alpha_a^2 \left(1 - \frac{v_a}{\kappa_a} \right)^2 + \beta_a^2} \right], \ \beta_a = \frac{2\alpha_a - 1}{2\alpha_a - 2}, \ \alpha_a > 1 \tag{14}$$

Akcelik (1991) proposes an alternative family of VDF that explicitly accounts for the delays incurred by traffic lights at signalized intersections:

$$c_a(v_a) = t_0^a \left\{ 1 + 0.25 \frac{T}{t_0^a} \left[\frac{v_a}{\kappa_a} - 1 + \sqrt{\left(\frac{v_a}{\kappa_a} - 1 \right)^2 + \frac{8J_a}{\kappa_a T} . \frac{v_a}{\kappa_a}} \right] \right\} \tag{15}$$

where T is the duration of the calibration interval, and $J_a$is the delay factor for link a, as defined by:

$$J_a = \frac{d}{1+\kappa_a T}$$

where d is a delay parameter whose value is d = 1 for exponential arrivals, and d = 0.5 for uniform arrivals. In all cases, the proposed VDF depends on parameters that must be calibrated, which requires observational data (Petrik, Moura and Abreu e Silva, 2014; Bessa, de Magalhães and Santos, 2021).

One common characteristic of all the proposed VDFs is that they are convex. Consequently, the symmetric static traffic assignment problem in user equilibrium, defined by (11), is a non-linear convex optimization problem. The properties of the convexity can provide benefits for the algorithmic approaches to solving the problem numerically.

Although the traffic assignment problem is a specific case of the non-linear multicommodity network flows problem and can be solved by any of the methods used for solving such problems, more efficient algorithms have been developed (LeBlanc, Morlok and Pierskalla, 1975; Florian and Nguyen, 1976) by adapting the linear approximation method of Frank and Wolfe (1956). Other efficient algorithms (Hearn, Lawphonpanich and Ventura, 1987; Lawphongpanich and Hearn, 1984) are based on the restricted simplicial approach, which exploits the properties of the convex polyhedron of feasible solutions defined by the constraints outlined in equation (10). Additionally, the parallel tangents method (PARTAN) introduced by Florian, Guelat and Spiess (1987) has proven to be effective. This version of the model is preferred by the main professional software platforms for transport planning because of its algorithmic ability to computationally deal efficiently with large road networks, particularly those found in large metropolitan areas. Consequently, research efforts have focused on improving these algorithms for solving the problem. Other contributions include those analytical approaches inspired by the gradient projection methods of Rosen (1960), which exploit the properties of the polytope defined by constraints (4) and combine them with the efficient shortest paths algorithms. Notable examples of these contributions can be found in Bar-Gera (2002), Dial (2006), and Florian and Constantin (2009).

Criticism of the symmetric traffic assignment problems arises from their inability to properly answer more demanding modeling requirements, due to the inaccuracies induced by the oversimplified separability assumptions in the VDF. This may happen, for instance, when dealing explicitly with delays at unsignalized intersections or when the generalized costs in multiclass planning models depend on vehicle class interactions that induce asymmetries. To address these issues, is necessary to develop alternative formulations that account for the asymmetries in either the space of the path flows (as in equations (5) and (6)) or in the space of the link flows (as in equations (9) and (10)). These models, known as asymmetric traffic assignment (ATA) in user equilibrium, are more appropriate for tackling the problem.

Equations (5), (6), (9), and (10) are typical variational inequality (VI) formulations in finite dimension spaces that correspond to an equilibrium principle. In essence, they can be described as follows (Codina, Ibáñez and Barceló, 2015): Given a closed convex set $X \in \Re^n$ of candidate solutions in which a continuous function $F(\cdot) : X \rightarrow \Re^n$ is defined, we look for a special point x such that the projection $x - \alpha F(x)$, for any $\alpha > 0$, onto X results in the point x itself. In other words, if $P_X[\cdot]$ is the projection operator on X defined by

$$P_X[z] = \text{argmin}\left\{\left(\|z-x\|_2^2 \middle| x \in X\right)\right\}$$

then a solution x of the VI verifies the fixed-point relationship

$$x = P_X\left[x - F(x)\right]$$

which is equivalent to stating that the solutions to VI problems satisfies the condition

$$F(x)^T(y-x) \geq 0,\ \forall y \in X$$

Rewriting (7) in vector form, we have $v = \Delta X$, and the VI (9) can thus be rewritten (Florian and Hearn, 1995) as:

$$C(\Delta X^*)\left[\Delta X - \Delta X^*\right] \geq 0,\ X \in \aleph$$

That is

$$\Delta^T C(\Delta X^*)(X - X^*) \geq 0,\ X \in \aleph \tag{16}$$

which can be solved by the projection algorithm:

$$X^{l+1} = P_{Q,\aleph}\left[X^l - \rho Q^{-1}\Delta^T C\left(\Delta^T X^l\right)\right]$$

and is equivalent to the convex optimization problem:

$$\text{Min}_{X\in\aleph}\left(X - X^l\right)\overline{C}(X) + \frac{1}{2\rho}\left(X - X^l\right)Q\left(X - X^l\right) \tag{17}$$

With $\overline{C}(X) = \Delta^T C\left(\Delta X^l\right)$, and Q a block-diagonal symmetric definite positive matrix (Codina et al., 2015), we have:

$$Q = \text{diag}\left[\ldots Q^{rs}\ldots;\ (r,s) \in I\right]$$

With each block Q^{rs} corresponding to an OD pair (r,s):

$$Q^{rs} = \text{diag}\left(\ldots q^{rsp}\ldots; p \in K_{rs}\right)$$

Then, (17) can be decomposed into the sequence of quadratic optimization problems, one for each OD pair:

$$\text{Min}\sum_{p\in K_{rs}}\left(x_{rsp} - x_{rsp}^{l-1}\right)\overline{C}_{rsp}\left(x_{rsp}^{l-1}\right) + \frac{1}{2\alpha_p^{-1}}\left(x_{rsp} - x_{rsp}^{l-1}\right)^2 \tag{18}$$

$$s.t.\ \sum_{p\in K_{rs}} x_{rsp} = X_{rs},\ x_{rsp} \geq 0$$

With $\alpha_p = \frac{\delta}{q^{rsp}}$ and δ a scaling parameter. Algorithms for numerically solving the subproblems proceed iteratively by generating new paths at each iteration while working in the sub-polytope of polytope $\aleph$ of the already identified paths and path flows. Codina et al. (2015) discuss the pros and cons of the alternative formulations of ATA in terms of VI, as well as the various algorithmic approaches for numerically solving them.

2.2. Dynamic traffic assignment models

With the emergence of intelligent transport systems (ITS), advanced traffic management systems (ATMS), and advanced traffic information systems (ATIS) as the most relevant ITS applications, planners need dynamic models that capture the time-dependent nature of changing traffic flows and traffic demand. The dynamic traffic assignment (DTA) problem can thus be considered an extension of the traffic assignment problem described above. DTA can determine time-varying links and path flows, and thus the temporal and spatial evolution of traffic flow patterns in the network (Mahmassani, 2001). The problem can be formulated as a dynamic user equilibrium problem built on the dynamic version of Wardrop's principle (Friesz et al., 1993; Smith, 1993; Ran and Boyce, 1996):

> If, for each OD pair at each instant of time, the actual travel times experienced by travelers departing at the same time are equal and minimal, the dynamic traffic flow over the network is in a state of travel-time-based dynamic user equilibrium (DUE).

Similarly to the translation of the static Wardrop's principle in into the variational inequalities (5) and (6), the DUE approach can also be implemented by solving the following mathematical model:

$$\begin{aligned} [tt_{rsp}(t) - \theta_{rs}(t)]\, x_{rsp}(t) = 0, &\qquad \forall p \in K_{rs}(t),\ \forall (r,s) \in I,\ t \in [0,T] \\ tt_{rsp}(t) - \theta_{rs}(t) \geq 0, &\qquad \forall p \in K_{rs}(t),\ \forall (r,s) \in I,\ t \in [0,T] \\ tt_{rsp}(t),\ \theta_{rs}(t),\ x_{rsp}(t) > 0, &\qquad \forall p \in K_{rs}(t),\ \forall (r,s) \in I,\ t \in [0,T] \end{aligned} \tag{19}$$

and the flow balancing equations

$$\sum_{\forall p \in K_{rs}(t)} x_{rsp}(t) = X_{rs}(t),\ \forall (r,s) \in I,\ t \in [0,T] \tag{20}$$

where, as before, $x_{rsp}(t)$ is the flow on path p from r to s, departing origin r at time interval t; $tt_{rsp}(t)$ is the actual path cost from r to s on route p at time interval t; θ_{rs} is the cost of the shortest path from r to s, departing from origin r at time interval t; $K_{rs}(t)$ is the set of all available paths from r to s at time interval t; I is the set of all origin-destination pairs (r,s) in the network, and $X_{rs}(t)$ is the demand (number of trips) from r to s, departing r at time interval t.

This is equivalent to solving a finite-dimensional variational inequality problem for finding a vector of path flows $\boldsymbol{x}^*$ and a vector of path travels times τ, such that:

$$[\mathbf{x} - \mathbf{x}^*]^{\mathsf{T}} \tau \geq 0,\ \forall \mathbf{x} \in \aleph \tag{21}$$

where $\aleph$ is the set of feasible flows defined by:

$$\aleph = \left\{ x_{rsp}(t) \middle| \sum_{\forall p \in K_{rs}(t)} x_{rsp}(t) = X_{rs}(t),\ \forall(r,s) \in I,\ t \in [0,T]\ \ ,\ x_{rsp}(t) > 0 \right\} \tag{22}$$

Wu et al. (1991), Wu, Chen and Florian (1998a), and Wu et al. (1998b), prove that this is equivalent to solving the discretized variational inequality:

$$\sum_{t \in [0,T]} \sum_{p \in \Re} tt_{rsp}(t) \left[x_{rsp}(t) - x^*_{rsp}(t) \right] \geq 0 \tag{23}$$

where $\Re = \bigcup_{(r,s) \in I} K_{rs}$ is the set of all available paths. This can be solved numerically with ad hoc projection algorithms, which are described in these references.

2.3. Models based on traffic flow theory

The approaches described so far are treat a trip as an analytical unit associated with an individual while also assuming that the two are separate and independent. Although path and link flows result from aggregating these trips, the proposed models implicitly ignore their nature.

However, an alternative hydrodynamic perspective views the temporal propagation of traffic flows as analogous to a fluid flowing through the network. This alternative modeling perspective aligns with Minsky's statement that a system can be modeled in different ways according to various approaches and the modeler's objectives.

This hydrodynamic analogy can be approached in two ways. One takes an aggregate perspective that focuses on the overall state of the fluid using aggregate macroscopic variables for density, volume, and speed. The other delves into the dynamics of the fluid by taking a fully disaggregated point of view that aims to describe the fluid process in terms of its constituent individual particle dynamics (the vehicles). Complete descriptions of these approaches can be found in Barceló (2010) or in Chapters 7 (Hydrodynamic and Kinematic Models of Traffic) and 6 (Car Following and Acceleration Noise) in the monograph *Traffic Flow Theory*, by Gerlough and Huber (1975).

Two independent papers published almost simultaneously (Lighthill and Whitham, 1955; Richards, 1956), introduced the fundamental principles of the hydrodynamic analogy for modeling traffic flows. This approach, known as the Lighthill–Whitham–Richards (LWR) model, considers a motorway with two counting stations, CS-1 and CS-2, as depicted in Figure 5, separated by a distance ΔX.

Let us first assume that traffic flows in the direction of the arrow, and that N_1 and N_2 represent the number of vehicles counted during the time interval Δt at CS-1 and CS-2, respectively. Then, if $N_1 > N_2$, there is an accumulation $N_2 - N_1 = \Delta N$ of cars between the two counting stations during time interval Δt, as there are no sources or sinks of cars in that segment.

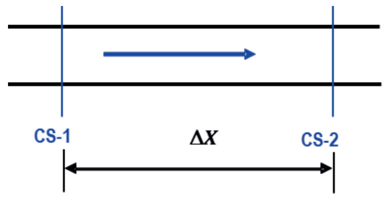

Figure 5.

The traffic flows are defined as volumes $q_1$and q_2 passing through counting stations CS-1 and CS-2 during time interval Δt, using:

$$\frac{N_1}{\Delta t} = q_1 \ \ and \ \frac{N_2}{\Delta t} = q_2 \tag{24}$$

Assuming that the density k (the number of cars per unit distance) is homogeneous in the space between the counting stations during the considered time interval, its variation over the distance between them is given by:

$$\Delta k = \frac{-(N_2 - N_1)}{\Delta x} = \frac{-\Delta N}{\Delta x}$$

and thus

$$\Delta k \cdot \Delta x = -\Delta N \tag{25}$$

Similarly, the flow variation $\Delta q = q_2 - q_1$ during time interval Δt while taking into account (24) will lead to

$$\Delta q \cdot \Delta t = \Delta N \tag{26}$$

Assuming the modeling hypothesis of the conservation of cars, that is the flow conservation, it follows from (25) and (26) that

$$\frac{\Delta q}{\Delta x} + \frac{\Delta k}{\Delta t} = 0 \tag{27}$$

Since the medium can be considered as a continuum, then we can take infinitesimal intervals to express equation (27) as:

$$\frac{\partial q}{\partial x} + \frac{\partial k}{\partial t} = 0 \tag{28}$$

which is the continuity equation for a fluid.

A simple model for a highway stretch splits it into contiguous sections and models the dynamics of the traffic flow in each one using equation (28). The resulting discretized model, shown in Figure 6, discretizes the traffic state variables–density k and flow q–in space and time, such that $k_i^j$and q_i^jrepresent, respectively, vehicle density per kilometer and vehicle flow per hour for cell i at instant j. The upper and lower rows respectively describe the states of the discretized cells at two consecutive instants, j and $j+1$. The

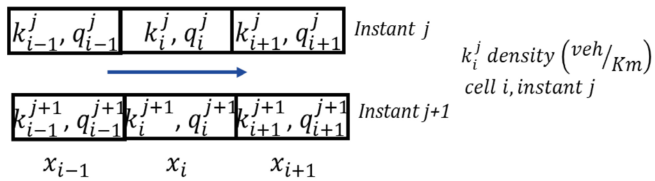

Figure 6. *Discretization of model (28).*

flow balance in cell i at time $j+1$, assuming that the inflow is equal to the outflow, can be expressed as:

$$k_i^{j+1}\Delta x = k_i^j \Delta x + q_{i-1}^j \Delta t - q_i^j \Delta t \tag{29}$$

This simpler model can be extended to highway sections with on- and off-ramps, like the one in Figure 7. These ramps serve as sources of incoming and outgoing traffic flows that are characterized by, respectively, rates $r(t)$ and $s(t)$. Thus, the continuity equation (28) (Michalopoulos, Beskos and Lin, 1984) becomes:

$$\frac{\partial q}{\partial x} + \frac{\partial k}{\partial t} = r(t) - s(t) = g(x,t) \tag{30}$$

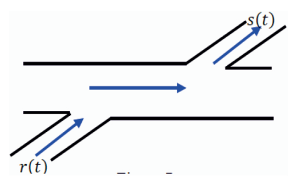

Figure 7.

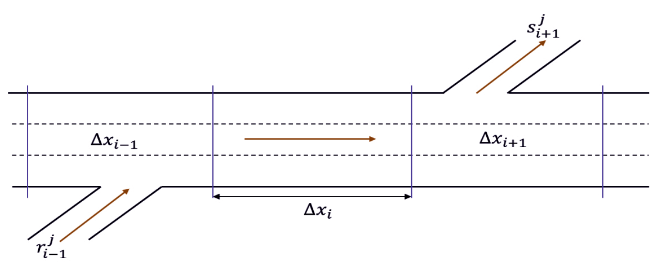

Figure 8. *Discretization to numerically solve the continuum motorway model.*

Defining $g(x,t)$ as the generation/dissipation function that balances the entry/exit flows, Michalopoulos (1984) generalized this model for motorways, as shown in Figure 8.

The time-space discretization of the model can be represented by the following set of difference equations:

$$k_i^{j+1} = \frac{1}{2}\left(k_{i+1}^j + k_{i-1}^j\right) - \frac{\Delta t}{2\Delta x}\left(q_{i+1}^j - q_{i-1}^j\right) + \frac{\Delta t}{2\Delta x}\left(g_{i+1}^j - g_{i-1}^j\right), \ \forall i \in I \tag{31}$$

where I is the set of cells.

This discretized form of the conservation equation is completed by an equation relating flows and densities, usually taking the form:

$$q_i^j = Q_i\left(k_i^j\right) \tag{32}$$

The set of equations (31) and (32) is usually known as the first-order macroscopic traffic flow model. For stability reasons, the time-space discretization in these models must satisfy the condition $\Delta x_i > u_f \Delta t_i$.

The interest in investigating the relationships between flows and densities arise simultaneously with the development of early traffic flow theories. These relationships came to be better understood by both measurement-based empirical evidence and theoretical analysis, such as Edie's (1963) seminal work. Let us assume a time-space diagram like the one depicted in Figure 9, where the traffic flows in spatially and homogenous conditions. The blue lines represent the vehicle trajectories in these conditions.

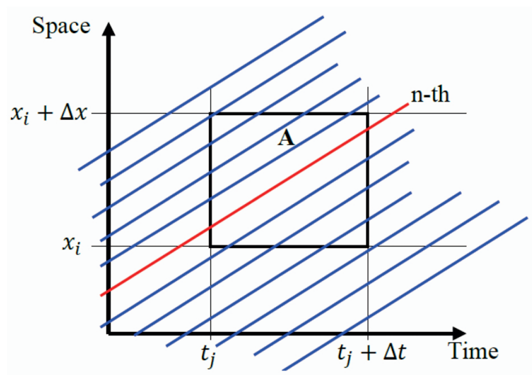

Figure 9. *Vehicle trajectories in the time-space diagram.*

Let us assume that one counting station is located at position x_i and the next counting station downstream is located at a distance Δx, corresponding to the location $x_i + \Delta x$. Let us also assume that Δt is the detection time resolution. Then, consecutive detectors and the detection time resolution define a discrete time-space Eulerian region A. Each vehicle (the n-th vehicle) follows a trajectory highlighted in red along a distance d_n during a time τ_n within this region. If $|A|$ is the surface of the region (in kilometers x hours), $d(A)$ is the total distance traveled by all vehicles crossing the region, and $t(A)$ represents the total time spent in the region by vehicles crossing it (in vehicles $\times$ hour). Defining the following quantities:

$$\text{Traffic Flow}\left(\frac{\text{vehicles}}{\text{hour}}\right) \qquad q(A) = \frac{d(A)}{|A|}$$

$$\text{Density}\left(\frac{\text{vehicles}}{\text{kilometer}}\right) \qquad k(A) = \frac{t(A)}{|A|}$$

$$\text{Mean spatial speed}\left(\frac{\text{kilometers}}{\text{hour}}\right) \qquad u(A) = \frac{d(A)}{t(A)}$$

the main relationship that emerges is $q(A) = u(A) \cdot k(A)$ or in the general form proposed by Daganzo (1997):

$$\mathbf{q} = \mathbf{uk} \tag{33}$$

which is known as the "fundamental traffic diagram". In the assumed stationary and homogeneous traffic conditions, it is further assumed that the mean spatial speed or equilibrium speed, denoted as $u_e(k)$ (Kotsialos and Papageorgiou, 2001) is a decreasing function of the density, which, combined with (33), leads to an equilibrium flow $q_e(k)$ that can be defined as:

$$q_e(k) = ku_e(k) \tag{34}$$

This corresponds to the steady state flow and homogeneous conditions. The fundamental diagram is a function that exhibits zeros at two extreme values of the density: $k = 0$ and $k = k_{\text{jam}}$, where the latter represents the jam density. This function has a unique maximum at the critical density $k = k_{cr}$, which corresponds to the maximum flow (capacity) q_{Max}.

Many mathematical expressions have been proposed for the function $u_e(k)$, based on either theoretical or empirical grounds. One of the most general is defined by the family of functions:

$$u_e(k) = u_f \left[1 - \left(\frac{k}{k_{\text{jam}}}\right)^{\alpha}\right]^{\beta} \tag{35}$$

where $u_f$is the free flow speed and α and β are parameters that must be calibrated. In this case, the complementary equations (32) of the first-order model for each section become:

$$q_i^j = k_i^j u_i^j \to \; u_i^j = u_f \left[1 - \left(\frac{k_i^j}{k_{\text{jam}}}\right)^{\alpha}\right]^{\beta} \tag{36}$$

Macroscopic models of traffic flows also provide additional examples of alternative models for the same system. First-order models can be extended to what is known as second-order models. These models, proposed by Papageorgiou and Kostsialos (2001) and TRB (2001), consider the mean speed as an independent variable and, therefore, require an extra equation for the speed dynamics, known as the momentum or speed equation. This introduces an additional modeling assumption that drivers respond to downstream traffic conditions with a corresponding reaction time, τ. Thus, the mean speed adjusts to the traffic density according to:

$$u(x,\, t+\tau) = u_e\left[k(x+\Delta x, t), t\right] \tag{37}$$

where Δx is the space increment. The expression (37) can be expanded using a Taylor series and, after rearranging the terms appropriately, we obtain:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{1}{\tau}\left[u_e(k) - u - \frac{\upsilon}{k}\frac{\partial k}{\partial x}\right] \tag{38}$$

where υ is a parameter whose value, according to Payne (1971), should be $\upsilon = -0.5\frac{\partial u_e}{\partial k}$. This parameter υ is usually interpreted in terms of viscosity. To solve equation (38) numerically, as with other models, it can be discretized in both time and space:

$$u_i^{j+1} = u_i^j + \frac{\Delta t}{\tau}\left\{u_e\left(k_i^j\right) - u_i^j + \frac{\Delta t}{\Delta x}u_i^j\left[u_{i-1}^j - u_i^j\right] - \frac{\upsilon\Delta t\left[k_{i+1}^j - k_i^j\right]}{\tau\Delta x\left[k_i^j + \kappa\right]}\right\} \tag{39}$$

Papageorgiou, Blosseville and Hadj-Salem (1990) propose including the term:

$$\frac{\delta\Delta t}{\Delta x}\left[\frac{r_i^j u_i^j}{k_i^j + \kappa}\right]$$

To model the impact of the entering on-ramp flows, δ and κ are additional model parameters whose values are estimated in the calibration process. The numerical solution to the second-order model is determined by equations (31), (36), and (39).

A variety of numerical methods for solving these macroscopic traffic flow models were developed between the 1970s and 1990s (Payne, 1971; Stock et al., 1973; Payne, 1979; Michalopoulos et al., 1984; Michalopoulos, 1984; Michalopoulos, Yi and Lyrintzis, 1993; Papageorgiou, Blosseville and Hadj-Salem, 1989; Papageorgiou, 1998).

2.4. Microscopic approaches

Continuing with the modeling exercise, we note here that transport and traffic systems offer fascinating examples of systems that can be modeled in various ways, depending on the objectives and underlying hypotheses. Let us now consider the traffic streams from a different perspective: a fully disaggregated standpoint aiming to explain the flow of the fluid in terms of its constituent individual particle dynamics. In the case of traffic flows, the individual particles are cars. One of the first attempts to describe flow dynamics in this way was the analysis conducted by Pipes (1953), who considered a line of traffic composed of n vehicles, as depicted in Figure 10, where L_k denotes the length of the k-th car and x_k its position. The modeling hypothesis is that the movement of several vehicles is controlled by a **law of separation**, which mandates that each vehicle must maintain a prescribed **following distance** from the preceding vehicle. This prescribed following distance, denoted as b, is proportional to the velocity of the following vehicle plus the minimum distance of separation when the vehicles are at rest.

Under this modeling hypothesis, the relationships between the positions of the consecutive vehicles, specifically the leader (vehicle k) and the follower (vehicle $k+1$) at time t are given by:

$$x_k(t) = x_{k+1}(t) + [b + Tv_{k+1}(t)] + L_k \tag{40}$$

where T is a time constant whose value is such that $Tv_{k+1}(t)$ satisfies the law of separation. From (40), the first derivative with respect to time provides the relationships between the speeds:

$$\dot{x}_k(t) = \dot{x}_{k+1}(t) + T\dot{v}_{k+1}(t) \rightarrow v_k(t) = v_{k+1}(t) + T\dot{v}_{k+1}(t) \tag{41}$$

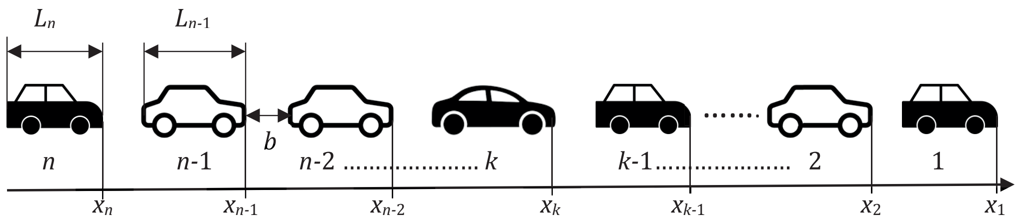

Figure 10. *Pipes' postulated line of traffic with n vehicles.*

which are the dynamical equations of the vehicle systems. Applying Laplace transform $\mathcal{L}$ and making $\mathcal{L}v_k(t) = V_k(p)$, this becomes the set of algebraic equations:

$$(Tp+1)V_{k+1}(p) = V_k(p) + Tpv_k(0) \tag{42}$$

If the velocity of the leader vehicle changes over time, then according to $v_1(t) = F(t)$, the velocity of vehicle k + 1 is given by:

$$v_{k+1}(t) = \left[\frac{T^{-k}}{(k-1)!}\right] \int_0^t u^{k-1} \exp\left(-\frac{u}{T}\right) F(t-u)du \tag{43}$$

These dynamical equations are based solely on physical considerations without taking into account that cars are driven by humans and are thus subject to certain conditions. This model tries to replicate the dynamic behavior of a vehicle stream, where vehicles follow one another and adjust their acceleration or deceleration in order to maintain a prescribed separation distance for safety reasons. However, no assumptions are made regarding how drivers and vehicles achieve this objective. Newell (1961) adopts a different perspective that will later be incorporated into a more generalized approach, in which empirical evidence leads to assuming a nonlinear relationship between a car's velocity at time t and the spatial headway a short time before (i.e., at time $t-\Delta$). Additionally, families of velocity–headway relationships align well with experimental data for steady flows. Building upon these observations, Chandler, Herman and Montroll (1958), Herman, Montroll and Potts (1959), and Gazis, Herman and Potts (1959) used empirical observations in the well-known General Motors experiments to lay the foundations of the car-following theory to model the dynamics of both vehicle and driver, which can be summarized as follows.

- The study of traffic is a combination of experimental and observational science.
- It adopts the perspective of the theory of servomechanisms, a branch of applied mathematics.
- It aims to clarify the role and interaction of the three components of the traffic system:
  - Road topology, which includes the number of lanes, nature of intersections, signals, warning signs, and other related factors.

– Vehicle characteristics, encompassing speed, acceleration and deceleration qualities, and other relevant attributes.
– Driver behavior, such as the range of perception and the lags between perception and response.
– In order to develop a theory of stable car-following, a traffic element should be considered as a servomechanism. The conceptual conditions for such a theory are displayed in Figure 11 (from Rothery, 2001).

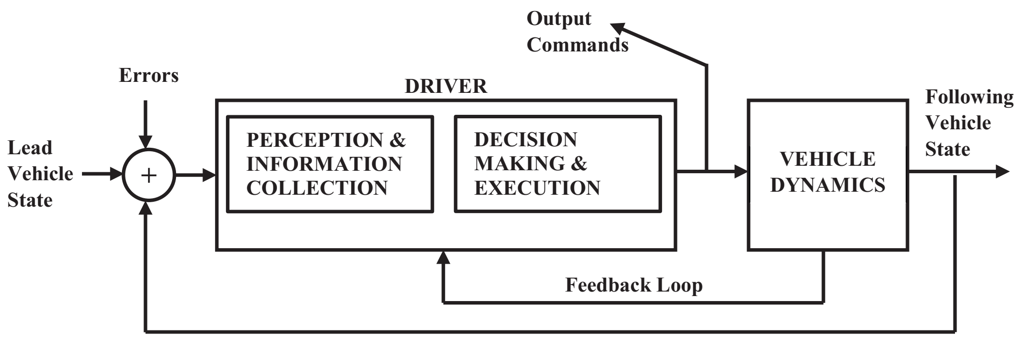

Figure 11. *Block diagram of the linear car-following model (Rothery, 2001).*

The main modeling hypotheses can be summarized as follows.

- Modeling assumption: Each driver reacts in a specific manner to stimuli from the preceding car(s), as stated by Gazis, Herman and Rothery (1961).
- Discrete cars move in continuous space and time.
- The laws of motion for each vehicle model driver behavior use differential-difference equations, which express the idea that each driver responds to a given stimulus according to a relationship defined by the following expression:

$$\text{RESPONSE} = \text{SENSITIVITY} \times \text{STIMULUS}$$

- The stimulus is a function of car positions, their time derivatives, and possibly other parameters.
- The response corresponds to the vehicle's acceleration or deceleration, as the driver has direct control through the gas and brake pedals.

Let us consider a situation like the one depicted in Figure 12, where various variables are defined as follows: x_n identifies the position of the *n-th* vehicle, $\dot{x}_n(t) = v_n(t)$ denotes its velocity at time t, and l_n its length. Additionally, $s_n(t) = x_{n-1}(t) - x_n(t)$ represents the spacing or space headway between the leader $n-1$ 1 and the follower n at time t, which is also called the effective length of vehicle n (vehicle length + safety distance). The relative velocity at time t between a leader-follower pair, i.e. n (leader) and n + 1 (follower), can be calculated as $\Delta v_{n+1}(t) = v_n(t) - v_{n+1}(t) = \dot{x}_n(t) - \dot{x}_{n+1}(t) = \dot{s}_{n+1}(t)$. The acceleration of vehicle n at time t is denoted by $a_n(t) = \ddot{x}_n(t)$.

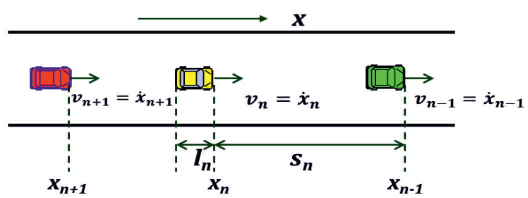

Figure 12. *Leader-Follower relationships.*

The car-following theory is based on the empirical observation that a strong correlation exists between a driver's response and the relative speed between their vehicle and the one ahead. The main modeling hypothesis is that the stimulus for a driver is the relative speed between each leader-follower pair, and the resulting mathematical model is a stimulus-response equation that describes the motion of the $(n+1)$-th car following the n-th car. In other words, the driver of the $(n+1)$-th vehicle observes variations in $v_n(t)$ or $s_n(t)$ and then accelerates or brakes to keep from lagging behind or getting too close to the leader. The mathematical model translating this hypothesis into formal terms is:

$$\frac{dv_{n+1}(t)}{dt} = F\left\{v_{n+1}(t); f_1\left[v_n(t) - v_{n+1}(t)\right]; s_{n+1}(t)\right\} \tag{44}$$
$$\frac{d^2x_{n+1}(t)}{dt^2} = \lambda\left[\frac{dx_n(t)}{dt} - \frac{dx_{n+1}(t)}{dt}\right]_{t-\tau}$$

Or, equivalently:

$$\ddot{x}_{n+1}(t+\tau) = \lambda\left[\dot{x}_n(t) - \dot{x}_{n+1}(t)\right] \tag{45}$$

This is a law for acceleration in a linear system, where λ represents the sensitivity of the control mechanism and τ is the time-lag of the driver-car system, which can be interpreted as the driver's reaction time. Integrating the model allows us to obtain the velocity of the n + 1-*th* vehicle, which represents the velocity of the traffic stream. Assuming that $s = x_n - x_{n+1}$ is the average spacing ($s = 1/k$) and when velocity $u = 0$, then spacing s_{jam} = jam spacing $= 1/k_{\text{jam}}$, and k_{jam} = jam density. The integration of equation (45) yields:

$$u = \lambda\left[\frac{1}{k} - \frac{1}{k_{\text{jam}}}\right] \quad and\ q = uk = \lambda\left[1 - \frac{k}{k_{\text{jam}}}\right] \tag{46}$$

which is the Greenshields, Gerlough, and Huber (1975) fundamental diagram of traffic. A detailed analysis of the model (44) reveals that the model is inconsistent with real data measurements. However, equation (46) indicates that it is conceptually consistent with the postulates of traffic flow theory.

Looking to improve the car-following model, Gazis et al. (1959) assume that the sensitivity λ varies with the distance between vehicles, as $\frac{\lambda}{s_{n+1}}$. In other words, the model takes into account that drivers' reactions will be quicker for denser traffic. Consequently, the updated model is as follows:

$$\ddot{x}_{n+1}(t+\tau) = \lambda \left[\frac{\dot{x}_n(t) - \dot{x}_{n+1}(t)}{x_n(t) - x_{n+1}(t)}\right] \tag{47}$$

which also exhibits inaccuracies. However, by integrating the equation once again to obtain the velocity of the n + 1-*th* vehicle (velocity of the traffic stream), and assuming that the stream velocity u is $u = 0$ for $k = k_{\text{jam}}$, the integration yields:

$$u = \lambda ln \left(\frac{k_{\text{jam}}}{k}\right) \tag{48}$$

That is Greenberg's fundamental diagram. Despite its inaccuracies, the car-following model (47) remains consistent with the fundamental principles of traffic flow theory. In seeking to improve car-following models, Edie (1963) proposed a modified model in which the sensitivity λ will depend on the square of the spacing and the current speed, resulting in:

$$\ddot{x}_{n+1}(t+\tau) = \lambda \frac{\dot{x}_{n+1}(t)}{[x_n(t) - x_{n+1}(t)]^2} [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \tag{49}$$

which can be integrated to give us

$$u = u_f \; exp\left(-\frac{k}{k_{\text{jam}}}\right) \tag{50}$$

It should be emphasized that the pursuit of more accurate car-following models is always rooted in the formal hypothesis of (44), which assumes that the follower's acceleration is a function of speeds, relative speeds, and spacings. This component of the model goes beyond physical considerations, since it tries to replicate human behavior, thereby increasing the complexity of the vehicle-driver system.

To conclude this non-exhaustive overview of car-following models based on the stimulus-response modeling hypothesis, two notable models that follow a similar trajectory are the Gazis-Herman general model and the Ahmed model. The Gazis-Herman model, proposed by Gazis et al. (1961) is given by:

$$\ddot{x}_{n+1}(t+\tau) = \lambda \frac{[\dot{x}_{n+1}(t)]^m}{[x_n(t) - x_{n+1}(t)]^l} [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \tag{51}$$

where l and m are parameters without physical meaning (in order to better fit the observations).

A generalized version of this model has been proposed by Ahmed (1999) and assumes an acceleration rate given by:

$$\ddot{x}_{n+1}(t+\tau) = \alpha^{\mp} \frac{\dot{x}_{n+1}^{\beta^{\mp}}}{g_{n+1}^{\gamma^{\mp}}} [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \tag{52}$$

These models assume different driver behaviors in following vehicles, depending on whether they are in the acceleration or braking phase. The model parameters $\alpha^+, \beta^+, \gamma^+$ are used for acceleration when $\dot{x}_n(t) \geq \dot{x}_{n+1}(t)$, and $\alpha^-, \beta^-, \gamma^-$ are used for deceleration when $\dot{x}_n(t) \leq \dot{x}_{n+1}(t)$. Here, l_n is the vehicle's length, and $g_{n+1} = x_n(t) - x_{n+1}(t) - l_n$ represents the gap distance from the leading vehicle (sometimes called the "effective distance").

In parallel to these developments other researches have sought a unified functional framework for car-following models. Newell (1961) explicitly expresses the dynamics in terms of a velocity-headway function:

$$\dot{x}_{n+1}(t) = V\left\{1 - exp\left[-\frac{\lambda}{V}\left(x_n(t) - x_{n+1}(t)\right) - d\right]\right\} \tag{53}$$

where λ and d are constants (calibration parameters) and V is the top velocity. Bando et al. (1995), Bando et. al (1998), and Treiber and Kesting (2013) propose a variant known as the "Optimal Velocity Model" (OVM), based on the dynamic equation:

$$\ddot{x}_{n+1}(t+\tau) = \alpha\left[V\left(s_{n+1}\right) - \dot{x}_{n+1}(t)\right] \tag{54}$$

Here, α is an acceleration constant, τ is a time-lag (which could represent the reaction time), and $V\left(s_{n+1}\right)$ is a velocity-headway function with s_{n+1} as the headway. One of the most widely used models within this family is the Intelligent Driver Model (IDM) by Kesting, Treiber and Helbing (2010):

$$\ddot{x}_{n+1}(t+\tau) = a\left\{1 - \left(\frac{\dot{x}_{n+1}(t)}{v_0}\right)^{\delta} - \left[\frac{s^*\left(\dot{x}_{n+1}(t), \Delta v_{n+1}(t)\right)}{s_{n+1}(t)}\right]^2\right\} \tag{55}$$

and

$$s^*\left(\dot{x}_{n+1}(t), \Delta v_{n+1}(t)\right) = s_0 + \dot{x}_{n+1}(t)T + \frac{\dot{x}_{n+1}(t)\Delta v_{n+1}(t)}{2\sqrt{ab}} \tag{56}$$

Here, T represents an anticipation time that also takes into account the velocity difference $\Delta v_{n+1}(t) = v_{n+1}(t) - v_n(t) = \dot{x}_{n+1}(t) - \dot{x}_n(t)$, that is, the approaching rate to the leading vehicle. The IDM combines a free flow acceleration function of the speed $a_f(v) = a\left[1 - \frac{v}{v_0}\right]^{\delta}$ with a braking strategy to decelerate $a_b = -a\left(\frac{s^*}{s}\right)^2$. This strategy becomes relevant when the gap between the follower and the leader is not significantly larger than the "effective gap" $s^*(v, \Delta v)$. The desired speed, v_0, is a behavioral parameter that differentiates drivers, and maximum acceleration is denoted by the parameter a, which also allows differentiating vehicles, meaning that the vehicles are not clones of each other. Finally, parameter δ characterizes how the acceleration changes with speed. Parameters a and b can be measured and calibrated (Kesting et al., 2010).

Ward and Wilson (2011) and Wilson (2011) formulate a common functional framework for car-following models, where a follower's reaction in terms of acceleration or deceleration depends on speeds, spacings, and relative speeds:

$$a_{n+1}(t) = \ddot{x}_{n+1}(t) = \mathcal{F}\left[s_{n+1}(t), \Delta v_{n+1}(t), v_{n+1}(t)\right] \tag{57}$$

These models have "uniform flow" steady solutions (equilibria) if, for each $s^* > l$, there is a $v^* = V(s^*) > 0$ such that $\mathcal{F}(s^*, 0, v^*) = 0$. Here, $V(s^*)$ represents the equilibrium speed-spacing relationship that leads to a fundamental diagram, thus ensuring that the car-following model is consistent with traffic flow theory. The general functional approach $\mathcal{F}$ serves a special interest by also laying the foundation for analyzing the stability of car-following models in terms of the partial derivatives, as outlined by Treiber and Kesting (2013). These derivatives must satisfy the conditions:

$$\mathcal{F}_s > 0,\ \mathcal{F}_{\Delta v} > 0,\ \mathcal{F}_v < 0 \tag{58}$$

To conclude this summary overview of these car-following models as alternatives to modeling the behavior of traffic flows, we will deal strictly with the main modeling aspects and discuss the family known as "collision avoidance" models (Gerlough and Huber, 1975; Barceló, 2010). These models assume that a follower driver will attempt to maintain a safety distance $s_n(t)$ from the lead vehicle, such that in the event of an emergency stop by the leader, the follower will come to a stop without colliding with the lead vehicle. The safe deceleration-to-stop diagram in Figure 13 (Gerlough and Huber, 1975; and Mahut, 1999) illustrate how this concept works.

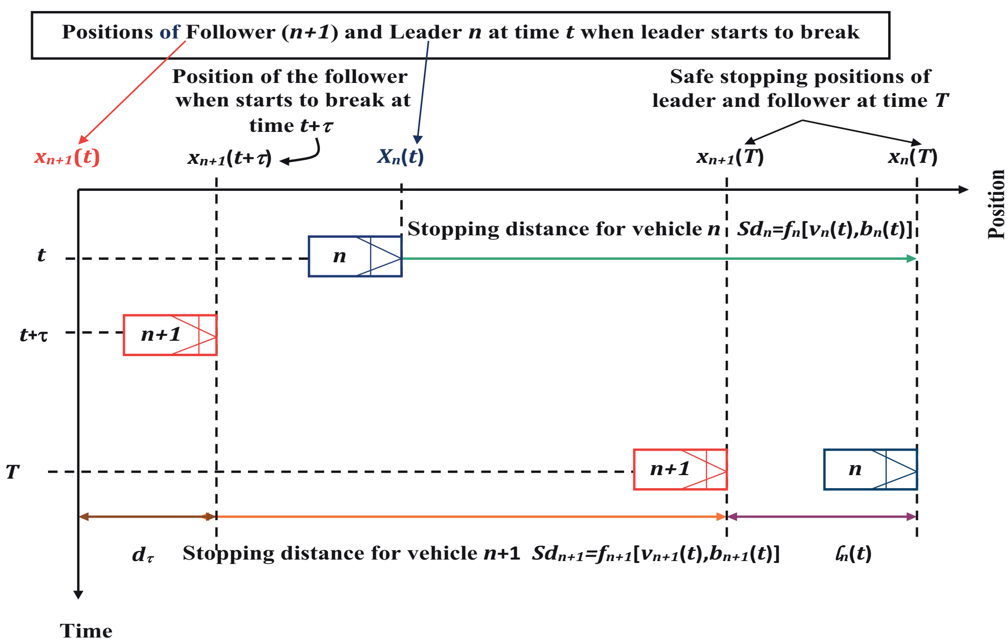

Figure 13. *Safe to stop diagram.*

This time-space diagram, as discussed in Barceló (2010), shows the positions of the leader n at time t, beginning with the initiation of braking until coming to a complete stop at time T. The follower $n+1$ perceives the leader's braking with a delay τ, representing the reaction time. During this delay, the follower travels a distance d_τ, which covers the position from the initiation of braking to coming to a safe stop. If Sd_n is the stopping distance for vehicle n, that is, the distance required to stop when traveling at speed $v_n(t) = \dot{x}_n(t)$ at time t, and the driver brakes with a deceleration function $b_n(t)$, then $Sd_n$is given by $Sd_n = f_n[v_n(t), b_n(t)]$. This function depends on the current speed $v_n(t)$ and the applied deceleration $b_n(t)$. Similarly, the stopping distance for the follower vehicle is given by $Sd_{n+1} = f_{n+1}[v_{n+1}(t), b_{n+1}(t)]$. Then, the desired spacing $s_n(t) = x_n(t) - x_{n+1}(t)$ at time t for a safe deceleration-to-stop is given by:

$$s_n(t) = x_n(t) - x_{n+1}(t) = \dot{x}_{\mathbf{n+1}}(t) \cdot \tau + Sd_{n+1}[v_{n+1}(t), b_{n+1}(t)] + \ell_n(t) - Sd_n(t)[v_n(t), b_n(t)] \tag{59}$$

where $d_\tau = \dot{x}_{\mathbf{n+1}}(t) \cdot \tau$ is the distance traveled by the follower during the reaction time τ, and $\ell_n(t)$ is the minimum safety distance (i.e., the distance between bumpers at rest). Assuming steady-state conditions with equal deceleration functions $b_n(t) = b_{n+1}(t)$, equal speeds, and, therefore, $Sd_{n+1} = Sd_n$, we have:

$$x_n(t) - x_{n+1}(t) = \dot{x}_{\mathbf{n+1}}(t+\tau) \cdot \tau + \ell_n(t)$$

Differentiating with respect to t, we obtain:

$$\dot{x}_n(t) - \dot{x}_{n+1}(t) = \tau \ddot{x}_{n+1}(t+\tau) \ \rightarrow \ \ddot{x}_{n+1}(t+\tau) = \frac{1}{\tau}[\dot{x}_n(t) - \dot{x}_{n+1}(t)]$$

which is the elementary form of the response to a stimulus model (45). Rewriting (59) as

$$x_n(t) + Sd_n(t)[v_n(t), b_n(t)] - \ell_n(t) \geq x_{n+1}(t)\dot{x}_{\mathbf{n+1}}(t) \cdot \tau + Sd_{n+1}[v_{n+1}(t), b_{n+1}(t)] \tag{60}$$

we can interpret this equation as a safety constraint that becomes active when it is satisfied as an equality, thus activating the braking action from the follower. Assuming steady-state conditions with $\ell_n(t) = \ell_n$ and constant deceleration functions $b_n(t)$ and $b_{n+1}(t)$ for a period, the respective distances to stop can be expressed as:

$$Sd_n(t) = -\frac{\dot{x}_n^2(t)}{2b_n} \ \ and \ \ Sd_{n+1}(t) = -\frac{\dot{x}_{n+1}^2(t+\tau)}{2b_{n+1}} \tag{61}$$

From equation (60), Mahut (1999) and Barceló (2010), we replace the Gipps speed of the follower during the deceleration phase, denoted as $v_{n+1}^b(t+\tau)$ (Gipps, 1981) and can thus be derived as:

$$v_{n+1}^b(t+\tau) = b_{n+1}\tau\sqrt{b_{n+1}^2\tau - b_{n+1}\left[2\{x_n(t) - \ell_n - x_{n+1}(t)\} - v_n(t)\tau - \frac{(v_n(t))^2}{b_n}\right]} \tag{62}$$

This includes a change proposed by Gipps (1981), which is based on empirical and consistency analyses. It replaces the leader's braking deceleration $b_n$with an estimated value $\widehat{b}_n$ because the follower does not have precise knowledge of the leader's deceleration. Furthermore, a safety margin is included to allow the follower a possible delay θ (e.g., $\theta = \frac{\tau}{2}$) when traveling at $v_{n+1}(t+\tau)$ before reacting to the braking of the vehicle ahead, thus satisfying:

$$-\frac{[v_{n+1}(t+\tau)]^2}{2b_n} + v_{n+1}(t+\tau)\left(\frac{\tau}{2}+\theta\right) - [x_n(t) - \ell_n - x_{n+1}(t)] + \frac{v_{n+1}(t)\tau}{2} + \frac{(v_n(t))^2}{2\widehat{b}_n} \leq 0$$

The so-called microscopic traffic models described in Barceló (2010) aim to replicate the propagation of traffic flows in a road network. The process, known as network loading, models the dynamics of car-following, lane-changing, and gap acceptance as vehicles travel from origins to destinations following route choice algorithms that mimic drivers' decisions. Microscopic traffic models have an advantage over continuum flow models because they handle traffic flow interruptions naturally, since car-following explicitly accounts for the possibility of the leader stopping. Consequently, these models can explicitly include traffic lights at signalized intersections to accurately represent the detailed phasing and timings. Figure 14 provides a graphical summary of the model-building process and its components.

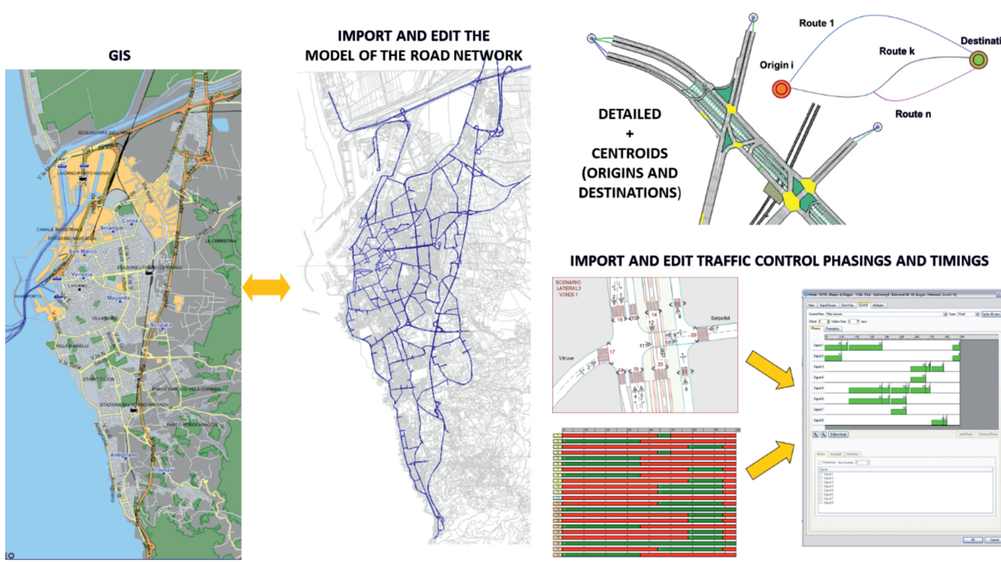

Figure 14. *Scheme of the microscopic model building process and the model components.*

2.5. Mesoscopic traffic models

DTA models have always been appealing due to two key characteristics. Firstly, they can handle large road networks, similar to the static assignment models used in transport planning. Secondly, they can account for time variations in transport demand and their impacts on the road network. However, their initial analytical approaches to solving the DTA formulation in (21), (22), and (23) proved to be computationally challenging. This generated interest in exploring alternative traffic simulation-based approaches that can provide approximate heuristic solutions. Florian, Mahut and Tremblay (2001) and Florian, Mahut and Tremblay (2002) proposed a conceptual framework that integrates both analytical and simulation-based approaches, as illustrated in Figure 15 (Barceló, Ros-Roca and Montero, 2022). The framework consists of two main interdependent components:

- A method for determining path-dependent flow rates on the network paths, which can be approached using various algorithms, ranging from the exact projection methods mentioned earlier to approximations like the Method of Successive Averages.

- A Dynamic Network Loading (DNL) method, which determines how these path flows translate into time-dependent arc volumes, arc travel times, and path travel times.

In the most successful practical implementations, the DNL method is usually based on a mesoscopic simulation model (Barceló, 2010) that emulates the flow propagation through the network under the current conditions. The resulting assignment depends on how the convergence criterion and iterative process are implemented. It can be a DTA or a dynamic user equilibrium (DUE) (Chiu et al., 2011). A mesoscopic traffic simulation model of traffic flow dynamics is a simplified representation that captures key aspects of microscopic simulation while being less data demanding and computationally more efficient than microscopic models. Mesoscopic approaches combine microscopic and macroscopic aspects of traffic flow dynamics, providing an alternative approach depending on the modeling objectives and hypotheses. In this paper, we will focus on approaches where flow dynamics are determined by the simplified dynamics of individual vehicles.

One such approach is the *cell transmission model* proposed by Daganzo (1994) and (1995a), which solves an ad hoc version of the first order traffic flow model using a simplified flow-density relationship known as the triangular (or trapezoidal) fundamental diagram (Daganzo, 1995b).

This basic model and its many variants, although widely used, exhibit limitations, namely in the case of urban networks, since they only account for flow dynamics in links and do not explicitly deal with intersections, most notably the signalized intersections commonly found in urban networks. To overcome these drawbacks, various extensions have been proposed to incorporate intersection modeling. One notable example is the *general link transmission model* (GLTM) developed by Gentile (2010) and Gentile (2015).

Another modeling alternative involves splitting the link into two parts, as shown in Figure 16. The first is the running part, where vehicles are not yet delayed by the queue spillback at the downstream node. The second is the queueing part, where the capacity is

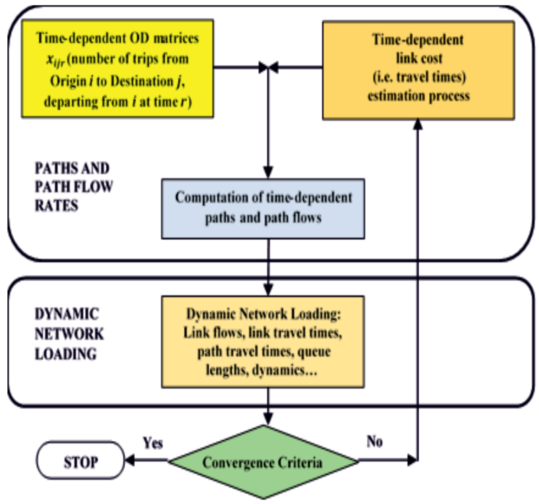

Figure 15. *Conceptual algorithmic framework for DTA.*

limited by stop signs, give-way signs, or traffic lights. Nodes are the interactions between traffic flows at intersections, and they can be modeled using either node transfer modules or a queue server approach that explicitly considers traffic lights and the delays they cause (Mahmassani et al., 1994). The flow dynamics in the running part are simplified in terms of macroscopic speed-density relationships by using variants of (36), such as:

$$u_i^t = (u_f - u_0)\left(1 - \frac{k_i^t}{k_{\text{jam}}}\right)^{\alpha} + u_0 \tag{63}$$

This equation, proposed by Jayakrisham, Mahmassani and Yu (1994), relates the mean speed u_i^t and density k_i^t in section i at time step t. The parameters u_f and u_0 are the mean free speed and the minimum speed, k_{jam} is the jam density, and α is a parameter that captures speed sensitivity to density. Alternatively, Ben-Akiva et al., (2001), and (2010), propose the following speed-density relationship:

$$u = \begin{cases} u_f & \text{if } k < k_{\text{Min}} \\ u_f\left[1 - \left(\frac{k - k_{\text{Min}}}{k_{\text{jam}}}\right)^{\alpha}\right]^{\beta} & \text{otherwise} \end{cases} \tag{64}$$

including a lower bound limiting density, k_{Min}, and a second parameter β to capture speed sensitivity to concentration. Vehicle dynamics in the queueing part are then governed by the queue discharging process. The boundary between the running part and the queueing part is dynamic and varies according to the queue spillback and queue discharge processes.

A completely different approach is taken by Mahut and Florian (2010), who propose a simulation model that moves vehicles individually using a simplified car-following model. In this model, the position $x_{n+1}(t)$ of the follower vehicle $(n+1)$ at time t relative

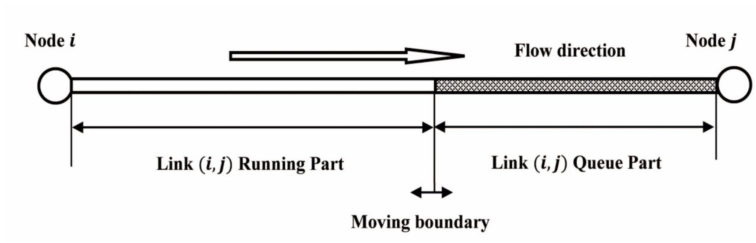

Figure 16. *Link model.*

to the position $x_n(t-T)$ of the leader vehicle (n) is estimated according to:

$$x_{n+1}(t) = \text{Min}\left[x_{n+1}(t-\varepsilon)+\varepsilon u_f, x_n(t-T)-l_{eff}\right] \tag{65}$$

where T is the reaction time, u_f the free-flow speed, l_{eff}, is the effective vehicle length, and ε is an arbitrarily short time interval. The first term inside the minimizing operator represents the farthest position downstream that the vehicle can reach at time t based on the follower's position at time $(t-\varepsilon)$ under the constraints of the maximum speed u_f. The second term inside this operator represents the farthest downstream position that can be attained based on the trajectory of the next vehicle downstream in the same lane, according to a simple collision-avoidance rule proposed by Mahut (1999) and Mahut (2001). This simplified model depends only on the free-flow speed and does not account for accelerations. It can be considered a lower-order model, since it determines the position of each vehicle in time, rather than their speed or acceleration.

The solution to the car-following relationship (65) for time can be expressed as:

$$t_{n+1}(x) = \text{Max}\left[t_{n+1}(x-\delta)+\frac{\delta}{u_f}, t_n\left(x+l_{eff}\right)+T\right] \tag{66}$$

This relationship allows for an event-based simulatiaon-4mm approach, as it enables calculating the link entrance and exit times for each vehicle by means of the following expression in Equation (67):

$$t_{n+1}\left(L_1\right) = \text{Max}\left[t_{n+1}(0)+\frac{L_1}{u_f^1},\ t_n\left(L_1\right)+T+\frac{l_{eff}}{\text{Min}\left[u_f^1, u_f^2\right]}, t_{n+\frac{L_2}{l_{eff}}}\left(L_2\right)+\frac{L_2}{l_{eff}}T\right] \tag{67}$$

where L_1 and L_2 are the lengths of two sequential links with speeds u_f^1 and u_f^2, respectively. The vehicle attributes l_{eff} and T are assumed to be identical throughout the entire traffic stream, and each vehicle adopts the link-specific free-flow speed when traversing a given link. The link lengths are assumed to be integer multiples of the vehicle length, l_{eff}. As shown by Mahut (2000), this model yields the triangular fundamental flow-density diagram proposed by Daganzo (1994). The main events that change the state of the model include vehicle arrivals at links, link departures, and transfers between links based on turning movements at intersections.

2.6. Models, valid models, and the data requirements for validating transport models

There is a common argument that emphasizes the abundance of data and suggests that models are becoming less necessary. Proponents of this view often strengthen their argument by quoting George Box's statement, "All models are wrong." Box, who is considered to one of the founding fathers of modern statistics and an expert in modeling recognizes, seemingly implies that models are useless. Although Box indeed made that assertion in 1976, the full context is often overlooked by those who quote him, as he later added an important caveat in the book by Box and Draper (1987), where he added "but some are useful," which, to the best of my knowledge, is frequently omitted.

Models as formal representations of systems are only approximations. As such, one should never forget that a representation of a system is not the actual system. Furthermore, when taking the Minsky triad perspective of building a model of a system, the modeler must ensure they are asking the right question and that the modeling hypotheses align with the objectives. As demonstrated in the previous sections discussing various modeling alternatives for traffic and transport systems, the goal is to highlight the diverse options available. Hence, considering Box's statement that some models are useful, the key question becomes: What makes a model useful?

The answer to that question is proper model validation and calibration, which are defined by Barceló (2010) and MULTITUDE (2014) as the following.

- **Calibration** is the process of determining model parameter values based on field data in a specific context. Parameters for the transport model in one city will differ from those for another city, and the nature of the parameters depends on the type of model and the objectives of the decision-making process supported by the model

- **Validation** aims to provide a quantitative answer to the question of whether model predictions faithfully represent reality. According to Rouphail and Sacks (2003), validation is determined formally by the probability that the difference between the "reality" and the "model prediction" falls within a tolerable difference threshold, denoted as d. This threshold measures the model's proximity to reality or, in other words, the error incurred when substituting the reality with the model. The level of assurance, denoted as a, measures the degree of certainty when making this substitution. The validation process satisfies the following criterion:

$$P\{|\text{reality} - \text{“model prediction”}| < d\} > a \tag{68}$$

It is the responsibility of the modeler conducting the study to define the criteria for model validation and acceptance. These criteria determine the values of parameters a and d, which assess the suitability and acceptability of the model. Figure 17 summarizes the methodological processes for calibrating and validating models, along with the data requirements for these processes. Looking at equation (68), in essence, the validation and calibration processes involve a statistical comparison between the observed "reality"

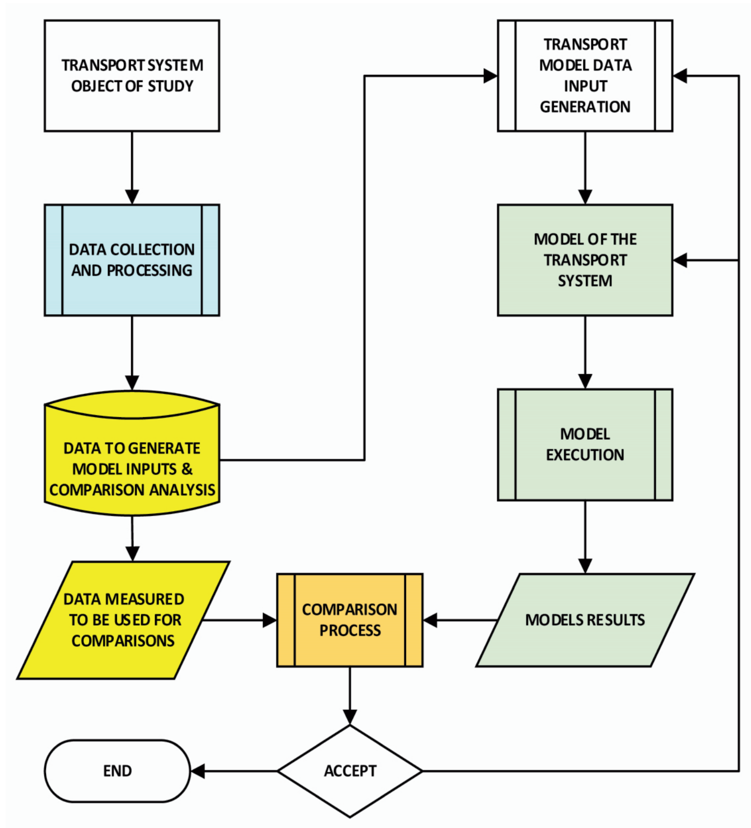

Figure 17. *Methodological scheme of model calibration and validation processes.*

and ”predicted” values of relevant variables that define the state of the system (e.g., speeds, flows, travel times). Usually, after identifying the variables to be used in the process, the corresponding data must be collected and processed using appropriate data analysis techniques. This includes cleaning the data, removing outliers and erroneous measurements, mitigating biases, and addressing missing values. Samples of observed data are usually split into independent subsets for various uses, such as calibrating model parameters, conducting statistical comparisons between observed and predicted data to validate the model, and, in some cases, generating inputs for the model.

All the models mentioned so far require the determination of parameter values and the calibration of data inputs, which are highly dependent on the specific system under study. Below are some examples.

- Static traffic assignment models:
  - BPR volume-delay functions for each link a (or class of links): t_0^a free flow travel time, κ_a link capacity, function parameters α_a, β_a
  - Conic functions, the additional delay factor J_a

- $\Delta = [\delta_{ap}]$ the link-path incidence matrix that depends on the network topology
- $X = [X_{rs}]$ number of trips from origin r to destination s for each OD pair (r, s)

- Dynamic assignment models
  - Time-dependent path cost functions $tt_{rsp}(t)$
  - Time-dependent (dynamic) OD matrices: $X(t) = [X_{rs}(t)]$

- Traffic flow theory models. The models described earlier involve various sets of parameters, depending on the order of the model:

- First-order models: the main parameters to calibrate determine the fundamental diagram, such as free flow speed u_f, the jam density k_{jam}, and the parameters *?* and *?* in equation (36).

- The second-order models in (38) will additionally require estimations of the equilibrium speed u_e and the viscosity v.

- Microscopic traffic models. These depend on a vast number of parameters and, as already mentioned, many aspects of the model-building process is automated in terms of importing road maps from GIS and setting the controls using original files, to name but two examples. For the purposes of this paper, let us focus here on car-following models and the parameters of the simulation engine:

- The Herman-Gazis car-following model (52): Parameters to fine-tune include values for the gap distance g_n between the leader and the follower as a function of the leader's length ℓ_n, as well as the acceleration and deceleration parameters $\alpha^{\mp}$, $\beta^{\mp}$, $\gamma^{\mp}$.

- The Gipps car-following models (62): Parameters to consider include deceleration rates b_n, reaction time τ, adjustment factor θ, and others.

It is worth noting that other car-following models, such as IDM (Kesting et al., 2010), the Wiedemann psycho-physical car-following model (Wiedemann, 1974), Fritzsche's car-following model (Fritzsche, 1994), or Krauss's car-following models (Krauss, Wagner and Gawron, 1997) are among the most widely used in the current traffic simulation software platforms.

Additionally, microscopic traffic simulation models, which involve vehicles travelling across the network from origins to destinations along paths, share certain requirements with DTA models. These include the calibration of time-dependent link travel times, time-dependent OD matrices $X(t) = [X_{rs}(t)]$, and path choice models that are typically based on discrete choice theory. Their utility functions may depend on factors like the value of time, which needs to be calibrated. Although controversial, path choice models are essential in capturing the behavioral aspects and topology of transportation modeling, especially in urban networks where the phenomenon of sharing links is common. To illustrate the nature of the problem, let us consider the following notation: $K_{rs}(t)$ is the set of paths from origin r to destination s at time t, $p(r,s,t)$ denotes the path $p \in K_{rs}(t)$, and $\Gamma_{p(r,s,t)} = \{e_1, \ldots, e_m\}$ is the set of links of path $p(r,s,t)$. If l_a is the length of link a, and $L_{p(r,s,t)}$ is the total length of path p, the commonality factor (Cascetta et al., 1996; Ben Akiva and Bierlaire, 1999) is a measure of the fraction of path p that is shared with all other paths h connecting origin r with destination s at time interval t. It is given by:

$$CF_{p(i.j.t)} = \frac{1}{\mu_{CF}} \sum_{a \in \Gamma_{p(,i,j,t)}} \left(\frac{l_a}{L_{p(i,j,t)}} \log \left(\sum_{h \in K_{rs(t)}} (\delta_{aht} + 1) \right) \right) \tag{69}$$

where δ_{aht} indicates whether link a also belongs to path $h \in K_{rs(t)}$ or not. The path choice proportion $P_{p(i,j,t)}$for each path on the set $K_{rs}(t)$ is calculated as a discrete choice model that uses the commonality factor within the OD pair and time. $CF_{p(r,s,t)}$ is a penalization factor added to current travel times (Bovy, Bekhor and Prato, 2008; Janmyr and Wadell, 2018):

$$P_{p(r,s,t)} = \frac{exp[\mu_{P_p}(-\widehat{tt}_{p(r,s,t)} - CF_{p(r,s,t)})]}{\sum_{h \in K_{rs(t)}} exp[\mu_{P_p}(-\widehat{tt}_{h(r,s,t)} - CF_{h(r,s,t)})]} \tag{70}$$

where $\widehat{tt}_{p(r,s,t)}$ is either the average travel time on the path $p \in K_{rs}(t)$ or the estimates of the average travel time value, and μ_{CF} and μ_{P_p} are parameters that must be calibrated.

- Mesoscopic traffic models depend on the set of values identified in equations (65)–(67), which must obviously be calibrated. Their relationships with the triangular fundamental diagram proposed by Daganzo (1994) must be explicitly considered. However, looking at the methodological computational scheme in Figure 15, we can see that they require inputs such as time-dependent OD matrices $X(t) = [X_{rs}(t)]$ and path choice functions similar to equations (68) and (69), which they have in common with the microscopic simulation models.

3. Data collection and analysis

The introductory remarks in this section emphasize the interdependence between data and models. It is essential to have data for the model-building process, and the overall model-building and utilization processes are illustrated in Figure 1. In Section 2.6, the conditions that render models useful are established through the model calibration and validation processes, which also need data, as depicted in the methodological scheme in Figure 17. Figure 18 summarizes a methodological approach that combines the key concepts outlined in Figures 1 and 17, incorporating elements from the OECD/ITF report (2015).

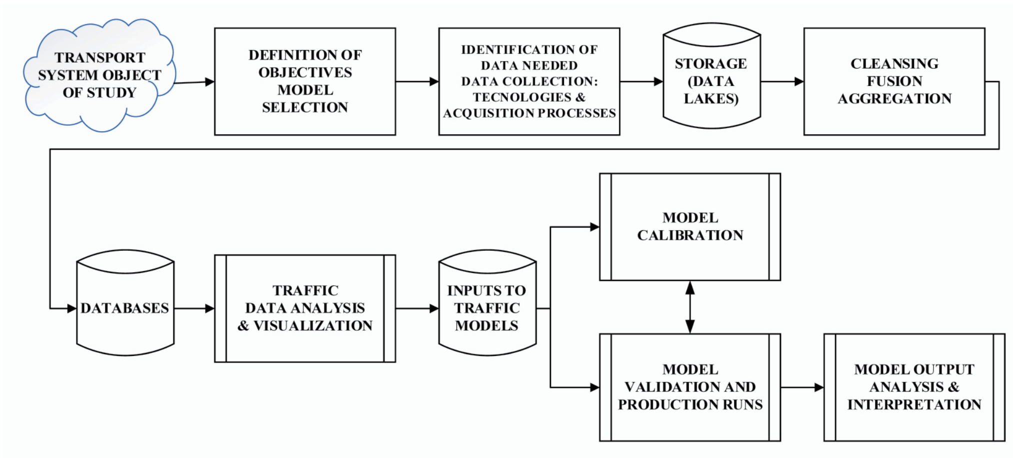

Figure 18. *Data collection and processing: A methodological approach for model construction, calibration, and validation.*

The steps in the process, which appear in the boxes of the methodological diagram are the following:

A. The first step, outlined in Section 1 of this paper, establishes a fundamental methodological principle for using models in analyzing transport systems. It emphasizes that different modeling approaches can be employed based on the objectives and associated modeling hypotheses. Therefore, the first step in using models to analyze transport systems is to identify the study objectives and determine the most suitable modeling approach to achieve them.

B. The second step, described in Section 2 of this paper, provides an overview of the most common modeling approaches for transport systems, depending on the objectives. These range from strategic approaches that employ either static or dynamic traffic assignment to the macro, meso, or micro approaches suitable for operational analysis or other purposes. Section 2.6 identifies the main parameters that must be calibrated for proper model building and use, along with the data needed for that process. Once the necessary data have been identified, the analysis must also ascertain the available technologies (Guerrero-Ibáñez, Zeadally and Contreras-Castillo, 2018) for data collection and determine the appropriate procedures.

B.1 Point detection with discrete time resolution: This includes inductive loop detectors, radars, etc., placed at specific positions, as depicted in Figure 19, which also indicates whether they are single or double loop detectors. They provide aggregated measures, with a Δt time resolution of:

- Average traffic flows in vehicles/hour

- Average occupancies: percentage of time a vehicle is over the detector with respect to the aggregation time Δt
- Average spot speeds (speeds measured at the detection point) in km/hour
- Traffic mix: percentages of light and heavy vehicles

B.2 Point detection with continuous time resolutions: This also occurs at specific positions, either on the road (e.g., inductive loops like magnetometers, as illustrated in Figure 19) or as roadside units (e.g., Bluetooth/Wi-Fi antennas, electronic TAG readers, CCTV image processing, etc.).

- Magnetometers measure the time in / time out of a vehicle passing over the detector and provide count flows, spot speeds, occupancies, and traffic mix, which can also be aggregated accordingly.
- Bluetooth/Wi-Fi and TAG readers identify the corresponding device onboard the vehicle and reidentify it downstream. They count only the flows of equipped vehicles (a non-representative sample of the whole population) and the time differences between two successive detection devices. Considering that the positions are well known and time differences are highly accurate, they provide a good sample of travel times or speeds between specific pairs of locations.
- CCTV image processing devices located at specific positions identify a vehicle by reading its license plate (license plate recognition) and reidentify it downstream. They have the advantage of detecting all vehicles, allowing measurement of point traffic flows and travel times between camera locations. If properly located, the cameras can also provide an estimate of OD matrices, with the origin being the point where the vehicle is first detected and the destination where it is last detected.

B.3 Continuous time-space detection, enabled by mobile devices that can be tracked along their trajectories, provides:

- In the case of GPS devices: waypoints with the detection time-tag; the vehicle's location (automatic vehicle location, AVL), consisting of the x, y and z coordinates (as shown in Figure 19); and, if the mobile device allows, the point speed at the detection time and heading.
- Mobile phones provide data from the call detail records, which can be processed to extract movements between origins and destinations. In some cases, inferences can be made about the routes used.
- Connected vehicles provide similar information about origins-destinations, travel times, speeds, locations, etc., either directly or via roadside units. In some cases, they can also provide additional similar data about the surrounding cars.

B.4 Public transport: Contactless cancellations provide a rich amount of data about passenger usage and transfers in public transport. Additionally, other ICT applications can provide passenger counting data aboard vehicles, etc. Bus monitoring systems provide detailed tracking information on schedule adherence, bus speeds, arrival times at stops, etc.

B.5 Shared vehicles: Data recorded by shared services vary depending on the company, type of vehicle (car, bike, etc.), and the equipment installed on the vehicle. Currently, there is no standard common type of data recorded. What is recorded may range from time and location of the service's origin and end, and in some cases a track of the route used.

Note: One subject of intense research has been optimizing the placement of point detectors, inductive loops, magnetometers, CCTV cameras, Bluetooth antennas, and other devices to provide measurements of partial path travel times, OD estimation, and others. This interesting optimization problem concerns the coverage problem in networks and the observability of the traffic system when measurements are used to estimate the system's state. Although the analysis of these models is beyond the scope of this paper, interested readers can find comprehensive overviews in Barceló et al. (2012) and Castillo et al. (2008).

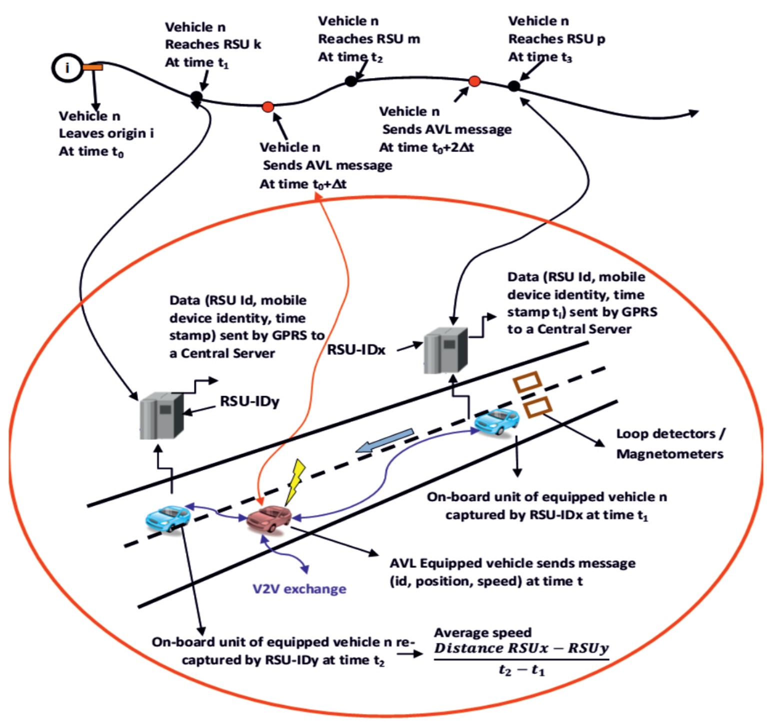

Figure 19. *Examples of data collection technologies.*

B.6 Other data sources

- Other ICT sources, such as social networks (e.g., WIZE, Moovit) or Google, can also provide data. However, these data often require specific treatment before being used in conjunction with other sources.

- Non-ICT networks: The summary overview of transportation modeling approaches highlights the key role played by traffic analysis zones (TAZ) and modal splitting in the four-step modeling approach. These models typically require socioeconomic and other data, such as information of transportation mode usage. No such data are currently available from TIC applications, although research efforts are actively addressing this gap. Furthermore, when dealing with transport mobility patterns represented by OD matrices, the usual ICT data sources only provide partial samples (e.g., from a subpopulation of vehicles equipped with a specific technology) that cannot be extrapolated to the whole population without complementary data. These complementary data sources are usually census tracts or carefully designed household surveys.

C. Third step: conventional data collection technologies such as those based on magnetic loops have historically provided limited and frequently scarce point observations at detection station locations. However, the emergence of new information and communication technologies (ICT) has dramatically changed the situation by granting access to large volumes of data, primarily spatial data, which necessitated proper storage using big data techniques (OECD/ITF, 2015). This vast and heterogeneous raw data is initially stored in unstructured data lakes, with an emphasis on acquiring it quickly, particularly for real-time operations.

D. Fourth step: Cleansing, fusion, and aggregation. Regardless of the quality of detection technologies, they are nevertheless prone to errors induced by temporal detection malfunctions or external factors affecting the detection quality (e.g., limited accuracy of GPS signals in certain urban areas). Therefore, the collected data must be properly cleansed before being used in transport models. This involves a series of data processes for identifying and filtering outliers to mitigate the risk of inducing undesirable biases, as well as completing missing data caused by outlier removal or lack of detection during a certain period. The objective of this step is to get clean, consistent, and complete data series.

   However, these clean and consistent data cannot be directly utilized since they originate from diverse data sources. For example, speeds measured by Bluetooth, CCTV cameras and inductive loop detectors may need to be fused. Similarly, generating modal split distributions may require combining data from mobile phones and household surveys. Data fusion techniques are employed to homogenize and harmonize these heterogeneous datasets to generate unique and consistent inputs.

Different models, depending on their specific use, may require different levels of data aggregation. For example, when dealing with OD matrices, time aggregation differs between static assignment models, dynamic assignment models, and dynamic traffic models. Therefore, depending on the model requirements, the data must be appropriately aggregated.

E. Fifth step: These clean, consistent, and structured datasets are then stored in databases that are specifically designed for data retrieval, tailored to the needs of different transport models.

F. Sixth step: Traffic data analysis and visualizations.

- This step involves generating the inputs required by the transport models, which will be elaborated upon in the subsequent sections.
- Additionally, advancements have been made in graphical techniques, which allow for data visualization and heat maps depicting degrees of congestion on network links or paths. These advances also highlight the attraction and generation capacities of the TAZs, such as in depicting travel patterns from origin to destination. Such descriptions of the system are useful for understanding the state of the system and assisting in the decision-making process.

G. Seventh step: Input to traffic models. The data for the transport model used in the study must formatted appropriately as input for the subsequent steps: calibration, validation and, once the model is validated, the production runs corresponding to the various scenarios to be analyzed and compared for decision-making purposes.

3.1. Notes on data cleaning

To illustrate some of the techniques used for data cleaning, including outlier removal, replacement of missing values, and correction of erroneous values, we will discuss two specific cases: the application of the Kalman filter to handle series of travel time measurements between two consecutive Bluetooth antennas; and using a map matching process to determine the correct position of a GPS waypoint within a network link.

3.1.1. Using a Kalman filter to clean time series of bluetooth travel time measurements

The Kalman filter, introduced by Kalman (1960) and further developed by Dailey, Harn and Lin (1996), is a state space model used to estimate the dynamics of a system. In this model, the state of the system at time instant k is defined by a set of unobservable state variables, represented by the vector $x_k \in R^p$ (where p is the number of state variables). The evolution of the system state transitions over time is governed by the linear stochastic equation in differences:

$$x_k = \Phi \cdot x_{k-1} + w_k \tag{71}$$

where Φ is the transition matrix and w_k represents the process noise, which is assumed to be white Gaussian noise with zero mean and covariance matrix Q. The system is observed at time k with measurements denoted as $y_k \in R^q$ (where q is the number of observations). The relationship between the measurements and the state variables is given by the linear measurement equation:

$$y_k = A \cdot x_k + v_k \tag{72}$$

where A is the measurement matrix with measurement noise v_k, which we also assume to be white Gaussian noise with zero mean and covariance matrix R. The process and measurement noises are assumed to be independent, with covariance matrices Q and R that can change at each step. The discrete Kalman filter cycles recursively between a temporal update and an estimation step. The temporal update projects the immediate future of the current state, and the covariance estimation provides an a priori estimate from steps k-1 to k, all by means of the following:

$$\begin{aligned} \widehat{x_k^-} &= \Phi \cdot \widehat{x}_{k-1} \\ P_k^- &= \Phi \cdot P_{k-1} \cdot \Phi^{\mathsf{T}} + Q \end{aligned} \tag{73}$$

The measurement update adjusts the projection of the estimate by incorporating the measurements available at that moment. It begins by calculating the Kalman gain, G_k, which is used to generate a posteriori estimates by incorporating the measurements $y_k$at that instant. The a posteriori estimate of the error covariance is also calculated:

$$\begin{aligned} G_k &= P_k^- \cdot A^{\mathsf{T}} \cdot \left(A \cdot P_k^- \cdot A^{\mathsf{T}} + R\right)^{-1} \\ \widehat{x}_k &= \widehat{x_k^-} + G_k \cdot \left(y_k - A \cdot \widehat{x_k^-}\right) \\ P_k &= (I - G_k \cdot A) \cdot P_k^- \end{aligned} \tag{74}$$

The process of filtering the observations of travel times, denoted as tt_j, is applied to the test day d (Barceló et al., 2010), as depicted in Figure 20. It uses the predictions $\widehat{x_k^-}$ and their variances, P_k^-, calculated by the Kalman filter (73) at each step k. This helps in selecting only valid observations, denoted as OV_d^k. From the valid observations, the implemented algorithm calculates the representative observations, y_k, for the current step k by applying the statistic $EST \in \{mean, median, ...\}$ to these observations:

$$\begin{aligned} OV_d^k &= \left\{ tt_{j \in O_d^k} \,\middle|\, \widehat{x_k^-} + \alpha \cdot \sqrt{P_k^-} \;\geq\; tt_j \;\geq\; \widehat{x_k^-} - \alpha \cdot \sqrt{P_k^-} \right\} \\ y_k &= EST(\, tt_{i \in OV_d^k} \,) \end{aligned} \tag{75}$$

Here, OV_d^k is a set of observations for the test day d obtained in the time interval k.

The Kalman filter uses the values y_k to calculate from the current state $\widehat{x}_k$, based on equation (74). This updated state estimate will be used in the subsequent predictions of (74) as part of the continuous filtering process. To filter the observations, limits have been calculated. These limits are derived from the Kalman filter's prediction by adding and subtracting α times the deviation considered in the same Kalman filter, thus obtaining the upper and lower limits.

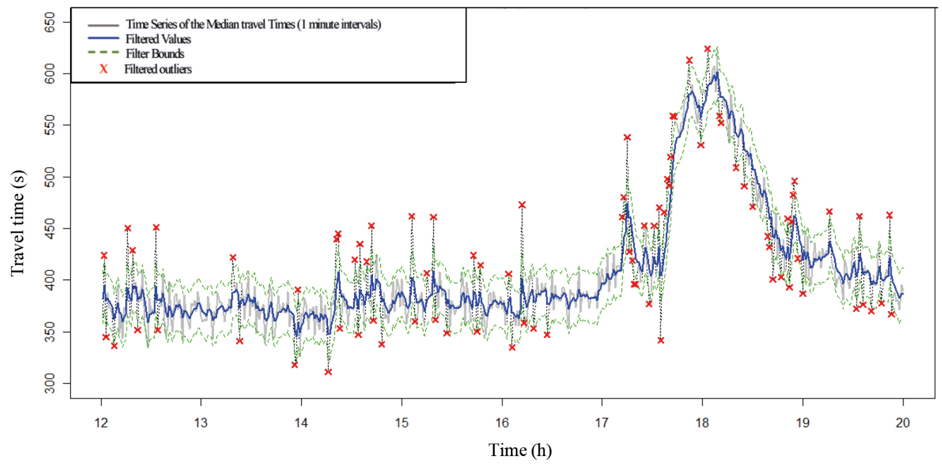

Figure 20. *An example of applying the Kalman filter to identify and remove outliers in 1-minute measurements of travel times between consecutive Bluetooth antennas on a specific test day. The outliers are then replaced with values that are consistent with observed data.*

3.1.2. Dealing with GPS data: map matching procedures

Commercial GPS data providers usually supply suitably processed data that is tailored to specific business models and applications. While the processing is a logical and natural part of their workflow, it often renders the data invalid for other general transport modeling approaches. However, unless the analyst is capable of designing their own data collection process and directly accessing the raw data, the most advantageous situation occurs when the transport analyst can obtain access to the waypoints generated by GPS. The left side of Figure 21 displays the most common and simple commercially available waypoints. Each waypoint consists of an arbitrary identity assigned by the provider to the mobile device in order to preserve the owner's identity, the date and timestamp of data collection, and the latitude and longitude corresponding to the tracked vehicle's position at that moment. The GPS data provider defines the collection policies and they can be collected at regular time intervals, after the vehicle has traveled a certain distance, at random times, and so on. The accuracy of GPS positioning can depend on various factors, such as the number of accessible satellites, signal intensity, whether the device is in an open or an urban area, and other variables. In urban areas, the accuracy is usually less than desired due to obstructions from buildings, signal interference, poor signal quality, and other factors. This can lead to erroneous positioning, as shown in the picture on the right side of Figure 21, where some waypoints are misplaced with respect to the network links, and a few may even be located on buildings.

Map-matching refers to the process of matching the geographical coordinates of waypoints to a model of the real world, such as a model of the road network in the case of tracking vehicles. The problem usually consists of relating the waypoints to the edges of the road network, which are provided by a geographic information system (GIS).

Due to its practical interest, map-matching is a widely studied problem. Kubicka et al. (2018) conducted a comparative study and application-oriented classification of selected vehicular map-matching methods covering the past two decades. The authors further provided guidelines for selecting a particular method, emphasizing that selection should be guided by the specific requirements of the application, distinguishing between offline and real-time applications. The particular case discussed in this section, as described by Cluet (2021), corresponds to a set of waypoints with high-rate positioning sampling, which will be processed using offline map-matching methods.

ID	Date	Timestamp	Latitude	Longitude
4261353	2019-11-30	07:43:58	45.445988	9.1244048
4261353	2019-11-30	07:44:11	45.445496	9.1241952
……….	…………	………	…………	…………
4261353	2019-11-30	07:45:08	45.444767	9.1192517
4261355	2019-11-30	07:45:02	45.445980	9.1247048
4261355	2019-11-30	07:45:23	45.445574	9.1192821
……….	…………	………	…………	…………
4261355	2019-11-30	07:46:56	45.444767	9.1197541

Figure 21. *Examples of waypoints and their misplacements.*

Geometric approaches were among the earliest used to solve the map-matching problem, due to the similarities between network link points and waypoints. A geometric map-matching algorithm uses the geometric information of the spatial road network data and primarily considers the shape of the links while disregarding their connectivity.

Given a trajectory *s,* geometric methods look for the most similar route in the map by using a shape similarity metric, δ. The most used similarity measures are based on distances (Hausdorff, Kim, and McLean, 2013; Fréchet, 1906), which aim to provide a good fit for the geometric aspect of the matching process.

The one-sided Hausdorff distance from curve *A* to curve *B,* as defined by Cluet (2021), is given by:

$$\delta_H^{'}(A,B) = \text{Max}_{a\in A}\,\text{Min}_{b\in B} d(a,b) \tag{76}$$

where $d(a,b)$ represents the Euclidean distance between points a and b. This is also known as the great circle or geodesic distance, which refers to the shortest distance between a and b on the Earth's surface. The Hausdorff distance δ_H is defined as the maximum value among the two one-sided Hausdorff distances:

$$\delta_H = \text{Max}\left[\delta_H^{'}(A,B), \delta_H^{'}(B,A)\right] \tag{77}$$

As pointed out by Kubicka et al. (2018), the Hausdorff distance has some shortcomings, such as failure to account for differences between routes that use the same road segment in opposite directions. In general, any two curves occupying the same area will have a small Hausdorff distance, even when they differ significantly in shape.

A more popular distance metric is the Fréchet distance, proposed by Fréchet in his 1906 thesis.

$$\delta_F(f,g) = \inf_{\alpha,\beta} \mathrm{Max}_{t\in[0,1]} d\left\{f\left[\alpha(t)\right], g\left[\beta(t)\right]\right\} \tag{78}$$

In this equation, f,g are parametrizations of curves, $f,g:[0,1]\rightarrow R^2$, while α,β are continuous, monotone, increasing parametrization functions $\alpha,\beta[0,1]\rightarrow[0,1]$. These parametrization functions are introduced to enforce continuous and monotonically increasing parameters for f and g.

Map matching methods based on both Hausdorff and Fréchet distance metrics are sensitive to outliers in GPS positioning observations

Probabilistic approaches aim to estimate the probability that a point belongs to a single segment. One of the most common approaches is to model it in terms of a hidden Markov chain, where the transition probability represents the likelihood of a point moving from one segment to another within a given time. In urban networks with complex topology, the geometric map-matching frequently fails to provide a unique road segment solution. Therefore, combining it with a probabilistic approach, such as the Viterbi algorithm (Viterbi, 1967) is often necessary. The Viterbi algorithm is a dynamic programming algorithm for obtaining the maximum a posteriori probability estimate of the most likely sequence of hidden states that results in a sequence of observed events, specifically in the context of hidden Markov models.

For the practical purposes of modeling the road network with OpenStreetMap (OSM) and PostGIS, there are several built-in functions available for calculating the distance between a waypoint and a link, which corresponds to the initial step in the geometric approach. This OSM and PostGIS computational environment also allows us to work with pgMapMatch, an open-source Python implementation of a map-matching algorithm developed by Millard-Ball, Hampshire and Weinberger (2019).

3.2. Fusing mobile phones, GIS data, and other sources for estimating OD matrices

3.2.1. Dealing with mobile phone data

Travel demand modeling, which encompasses travel patterns and transportation mode usage, has been traditionally conducted using household techniques, as discussed in Section 1 in the summary description of the four-step model. However, the emergence of mobile smartphones with location capabilities has led to the development of novel approaches based on mobile technologies. Among them is the idea of digital diaries, which enable recording people's behaviors in urban spaces by means of probe person technology. In their seminal paper, Asakura and Hato (2004) introduce the fundamental concepts and methodologies for using smartphones to conduct tracking surveys of individuals in urban areas. While these survey techniques allow for the collection of detailed trip and traveler information, they unfortunately suffer from drawbacks, such as limitations on sample size and requiring active participation from each individual in the sample. To overcome these drawbacks, Asakura and Hato (2009) propose additional technological developments aimed at passive data collection, later on explored by Hato (2010). Itoh and Hato (2013) suggest improvements in sampling techniques.

However, because the widespread adoption of mobile phones has made them ubiquitous and technological advancements has led to them becoming effective sensors, an alternative line of research has emerged, one which exploits *call detailed records* (CDR) of phone calls and text messages (SMS) exchanged between customers. These records are automatically collected by mobile phone service providers and offer a cost-effective and frequently updated source of data consisting of timestamps and antenna IDs. Call positions are identified according to the connected antenna, whose position is given as longitude and latitude. The time stamps can be aggregated into time intervals that align with the study's objectives. Ratti et al. (2006) were among the first to develop and utilize these techniques, and González, Hidalgo and Barabási (2008) conducted the first large-scale data evaluation of mobile phone data. Since then, this field of research has flourished in relation to the modeling and analysis of travel demand (Alexander et al., 2015; Toole et al., 2015). Moreover, the research has been applied to traffic analysis and transport models (Jiang et al., 2016; Çolak, Lima and González, 2016). These developments have progressed to the point where commercial products are now available and being exploited by companies for use in transportation projects (García-Albertos et al., 2018; Bassolas et al., 2019).

As with any other kind of observation, the huge amount of data recorded from CDR requires careful cleansing to filter out noise caused by errors in assigning mobile phones to cell towers, particularly during the tower-to-tower balancing performed by the mobile service provider. This crucial initial step is necessary to reliably extract activities and trips from CDR data. Many of the abovementioned papers dedicate specific sections to the wide variety of filtering procedures that can be applied. Subsequently, mobile phone trajectories are analyzed using data mining procedures to identify trips, represented by their start/end locations and departure times.

For such reliable inferences of activities and trips, we must distinguish between locations where users stay (where activities occur) and their moving pass-by locations (en route displacements). The conventional methods for making such distinctions are based on the agglomerative clustering algorithm proposed by Hariharan and Toyakama (2004). These methods identify points that are close in space but distant in time, along with additional criteria such as those proposed by Levinson and Kumar (1994), Schafer (2000), and Alexander et al. (2015). Essentially, these methods consist of identifying a stay point as a sequence of consecutive mobile phone records based on spatial and temporal thresholds. The spatial thresholds are set up in terms of a roaming distance, which is a parameter that must be calibrated according to network topology, phone cell density, signal quality, the accuracy of location positioning technology, and other relevant considerations.

The temporal thresholds are defined in terms of the minimum length of the stay time, which is a parameter that needs to be calibrated. This measure is calculated as the time difference between the timestamps of the first and last records at each stay point. Users' visited locations and stay points can be categorized into types such as home, work, frequently visited places (e.g., shopping centers), and infrequently visited places. This categorization can be based on using rationale and historical evidence to record stay durations during weekdays, such as being at home between 9 pm and 8 am and at work between 9 am and 6 pm (Jiang et al., 2016). Figure 22 presents a generic situation adapted from Jiang et al. (2016), and it shows that stay points can also be clustered into stay regions. For example, in the context of defining trips based on mobile phone observations provided by CDR, let us assume that movements start from the home location in the morning and end at the home location in the evening, unless the user's distance to the home exceeds a threshold value denoted as d_{Max}. A threshold value d_{Min} defines a minimum movement distance to identify successive records.

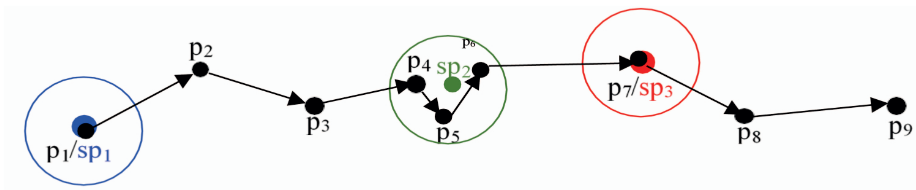

Figure 22. *p_i represents the i-th observation of a mobile phone, and st_j, denotes the j-th stay-point (home sp_1, shopping center sp_2, work place sp_3 ...). Mobile records p_4, p_5, and p_6 are clustered into stay point sp_2. Circles identify the thresholds.*

```
Algorithm 1:
main()
1   for each user u
2     for each day a
3       for each CDR observation k
4         let p_uak = position for event k
5           if(trip_active == false)
6             trip_active = detect_trip_start()
7             end
8           if(trip_active == true)
9             trip_ended = detect_trip_end()
10        end
11            if(trip_ended)
12        store_trip()
13        end
14      end
15    end
16  end
```

```
Algorithm 2
detect_trip_start()
17  if (trip_set empty)
18    if(p_uak != homebase and d(p_uak,homebase) < d_max
        and d(p_uak,homebase) > d_min)
19      trip_active = true
20      origin = homebase
21  end
22  if(p_uak != homebase and d(p_uak,homebase) > d_max)
23    trip_active = true
24    origin = p_uak
25  end
26  if(p_uak == workbase and d(p_uak,homebase) > d_max)
27    trip_active = true
28    origin = homebase
29    destination = workbase
30  end
31  else
32    if(p_uak != previous_trip_start(trip_set) and
      d(previous_trip_start(trip_set), p_uak) > d_min)
33    origin = previous_trip_start(trip_set)
34  end
35  end
```

```
Algorithm 3
detect_trip_end ()
36  if(p_uak == workbase or p_uak == homebase)
37    destination = p_uak
38    else
39   if(p_ua(k+1) exists)
40   if(p_uak == p_ua(k-1))
41   destination = p_uak
42   end
43  else
44   if(d(p_uak,homebase) < d_max)
45   destination = homebase
46   else
47   destination = p_uak
48   end
49  end
50  end
```

Exhibit 1. Trip Generation Algorithms

The trip generation algorithm (Gundelgård et al., 2015, 2016) shown in Exhibit 1 uses CDR observations and consists of three functions. The main function (Algorithm 1) is called *main*() and loops through all available CDR observations for each user and for each day. It scans each observation, invoking the *detect_trip_start*() function (Algorithm 2) to determine whether the trip start condition is met. This condition is satisfied if the distance from the ending point of the previous trip (line 33 of the pseudocode) or from the home position for the first trip of the day (line 19) exceeds the value d_{Max}. While a trip is in progress, the *detect_trip_end*() function (Algorithms 3) is invoked for every observation. The algorithm considers a trip has ended if the user arrives at home (line

37), work (line 45), or if two consecutive events have the same position (line 41). When a trip has ended, the *main()* function repeats the process and tries to detect the user's next trip start by calling *detect_trip_start()*.

Additionally, it is possible to extract stay locations with durations exceeding a certain threshold (a parameter defined by the analyst, depending on the context). If the observation period is sufficiently long, all frequently visited stay locations can be identified. Trips can also be filtered and aggregated into time intervals to estimate dynamic OD matrices. During these periods, origins and destinations can be estimated by identifying the most common positions and associating them with TAZs, particularly when the origin zone differs from the destination zone. Identifying the area of study presents another significant consideration when generating OD matrices from mobile phone data. Urban databases typically store socioeconomic, population, and other related data at the level of census tracts. The travel patterns defined by OD matrices correspond to traffic analysis zones (TAZ), as discussed in Section 1. Determining how to define TAZ splitting is not a trivial task and usually takes into account the socioeconomic characteristics of the studied region, which aligns with considerations about the underlying causes of mobility (Ortuzar Willunsen, 2011). TAZs are usually well-balanced in terms of population and demand analysis criteria, and they are frequently formed by aggregating census tracts through a clustering process that covers the entire territory. Finally, the cellular cells associated with each mobile phone antenna form the third layer covering the territory, which may not have a direct correspondence with the other two partitions covering the territory.

Consequently, in order to avoid significant errors caused by misalignments and inconsistencies between the three coverings, careful design is necessary (Zhang et al., 2010; Iqbal et al., 2014; Montero et al., 2019). For example, Bassolas et al. (2019) propose a heuristic to overcome the lack of exact correspondence between Voronoi cells. Their method assigns residents located in a given Voronoi cell to one of the intersecting census tracts or neighboring areas. The assignment probability is directly proportional to the square of the population of the census tract and inversely proportional to the square of the number of users already assigned to that tract. This assignment process ensures a local homogeneous sample density among neighboring census tracts. Figure 23 (adapted from Gundelgård et al., 2015, 2016) depicts an example of Voronoi tessellation modeling the phone cells and assignments of trips to TAZs. These TAZs are the result of aggregating Voronoi polygons based on mobile phone data for Senegal, which was obtained from the mobile operator Orange (de Montjoye et al., 2014). The data comprises call detail records (CDR) of phone calls and text exchanges (SMSs) between customers in Senegal, collected between January 1, 2013, and December 31, 2013.

The data used in Gundelgård et al. (2015, 2016) consists of 1666 antenna IDs and their corresponding locations, as well as mobility data for a year. The mobility data is based on a rolling two-week basis and comprises approximately 300,000 randomly sampled users.

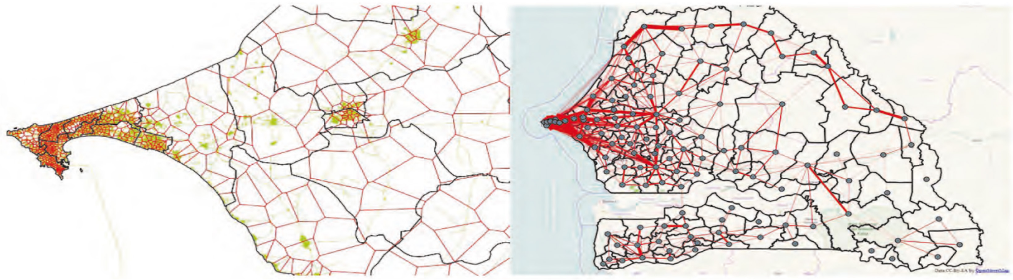

Figure 23. *Example of Voronoi tessellation and trip assignments to TAZ.*

Figure 24 summarizes the described methodological process. The upper part of the logic diagram represents the processing of the CDR. It is important to note that the proposed process produces a global OD matrix, which includes all trips regardless of the transportation mode used. In other words, there is no distinction between trips by car, bus, or any other public transport mode such as metro or railway. However, as discussed in Section 2 when describing the inputs to various models (particularly DTA and microscopic models), the required input is a dynamic time-dependent OD matrix $X(t) = [X_{rs}(t)]$ specific to each transport mode. For example, in the practical cases addressed in this paper, separate OD matrices are needed for car trips. Therefore, an additional step is needed to generate such OD matrices.

The most common solution to this problem is depicted in the conceptual diagram in the lower part of Figure 24. It consists of integrating the $\mathrm{OD}^{\mathrm{CDR}}$, which was initially estimated from the CDR processing, with other data sources that explicitly account for modal splitting. The most typical source is the household transport survey, which has long been used in transport demand analysis. Household surveys have the disadvantage of representing a small sample of the whole population and providing a kind of snapshot that is valid only for the time when the survey was conducted. However, they offer the advantage of being produced by carefully designed samples using well-established statistical sampling techniques that ensure being able to reliably extend the sample to the whole population. The fusion of this (possibly outdated) historical OD^{H} with the more accurate and updated $\mathrm{OD}^{\mathrm{CDR}}$ is then used to derive a set of mode-specific matrices ($\mathrm{OD}^{\mathrm{mode}}$). This can be achieved by establishing correspondences between the splitting rates of the initial OD and assuming that they will prevail in the second (Montero et al., 2019). Alternatively, historical data can be used to calibrate a discrete choice model (as discussed in Section 2.1) and apply it to the $\mathrm{OD}^{\mathrm{CDR}}$ to estimate the modal ODs.

Once a modal OD has been obtained, such as the $\mathrm{OD}^{\mathrm{car}}$ for car trips, it can be refined if additional data sources are available. The most usual case is when conventional traffic data, link flows, and speeds, are accessible available from the traffic management system operating in the corresponding area. The estimation of OD matrices from available traffic measurements to generate inputs to transport models is a notably problem that has garnered substantial attention from researchers. This attention has been driven by its relevance for practical applications, especially in recent years with the growing

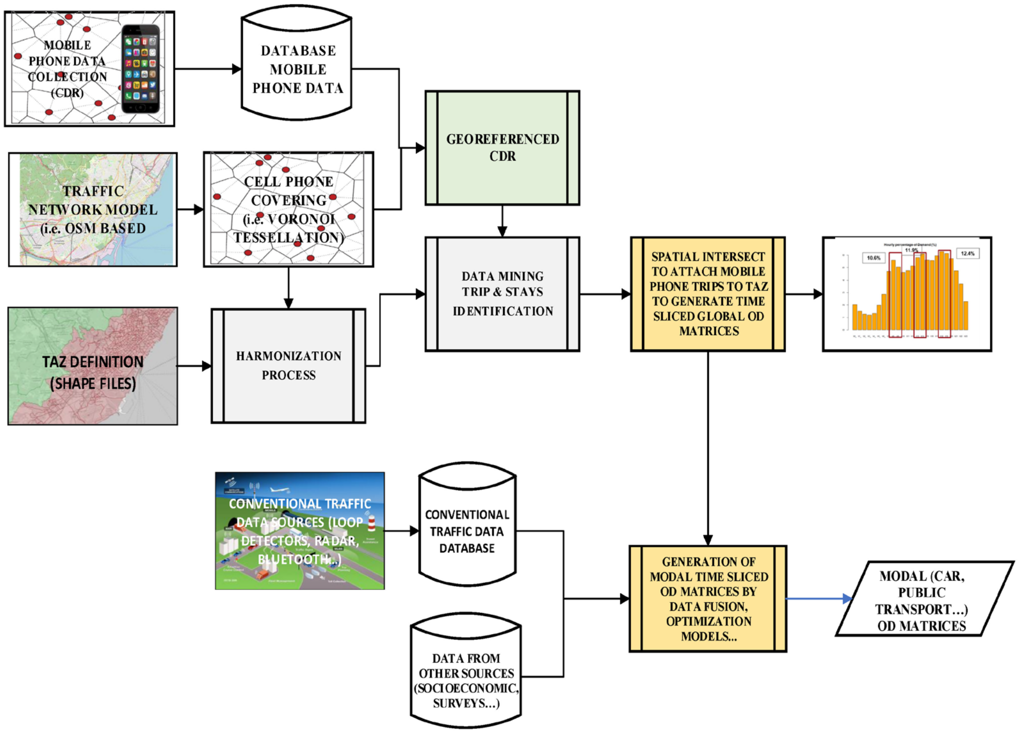

Figure 24. *Methodological diagram of the procedures for generating OD matrices from CDR.*

demand for dynamic models that require dynamic inputs (see Antoniou et al., 2016 for an overview).

The basic assumption is that, given an OD matrix X, an equilibrium assignment (as described in Sections 2.1 and 2.2) provides estimates of the link flows Y and, in some cases (e.g., dynamic assignments), estimates of other traffic variables such as paths, partial paths, and travel times. The reciprocal problem, as discussed by Cascetta (2001), can be formulated as follows: Assuming that $\widehat{Y}$ is a set of observed link flows in a subset of links in the network, the goal is to find the OD matrix $\widehat{X}$ whose equilibrium assignment onto the network will generate the observed flows. Mathematically, the problem is highly underdetermined and challenging, but acceptable solutions can be found by imposing additional constraints (Cascetta, 2001; Antoniou et al., 2016). It can be formulated as a nonlinear optimization problem in various forms, such as the following highly suitable bilevel optimization problem:

$$\begin{aligned} \text{Min } Z(X) &= \mathcal{F}\left(X, \widehat{X}, Y, \widehat{Y}\right) \\ s.t.\ \widehat{Y} &= \text{assignment}\left(\widehat{X}\right), \quad \widehat{X} \in \Omega \end{aligned} \tag{79}$$

where $\mathcal{F}$ is typically a distance function that measures the difference between a target or historical matrix X and the estimated matrix $\widehat{X}$, as well as between the observed link flows Y and the estimated flows $\widehat{Y}$. The feasibility dominion Ω is usually determined by additional constraints (Djukic, 2014; Antoniou et al., 2016). To enhance computational efficiency, the most efficient linearization approaches are often employed to approximate the assignment used in (79). This is especially true in dynamic cases, when the OD matrix is time-dependent and replaces the assignment mapping with:

$$y_{lt} = \sum_{ijr} a_{ijr}^{lt} x_{ijr} \;\rightarrow\; Y = A(X)X \tag{80}$$

Here, a_{ijr}^{lt} is known as the assignment matrix and represents the proportion of the OD flow departing from origin i at time r and going to destination j, crossing link l at time $t \geq r$. Various linearization approaches have been proposed in practice, such as those presented by Toledo and Kolechkina (2013), Frederix, Viti and Tampère (2013), and Ros-Roca et al. (2022). Several algorithmic approaches have been proposed to solve the model, such as those by Toledo et al. (2013), Antoniou et al. (2016), and Ros-Roca et al. (2022). There are variants of the simultaneous perturbation stochastic approximation (SPSA) method originally proposed by Spall (1992) that are explored in Antoniou et al. (2016) and Ros-Roca, Montero and Barceló (2020), among others. Another noteworthy approach is simulation-based optimization, as described by Osorio (2019), which is well suited to dynamic cases involving a simulation-based approach.

3.2.2. OD Estimation and GPS data

An emerging trend enabled by the accessibility of GPS traces is the development of so-called "data-driven" models in which the parameters of the mathematical model of the transport system are directly estimated from ICT measurements. These approaches rely on large samples of vehicle data collected over a sufficiently long period. The first step in the process, as discussed in Section 3.1.2, consists of obtaining individual vehicle trajectories from GPS records and map-matching these waypoints onto the graph of the transportation network by means of specialized map-matching algorithms suitable for the available sample, whether it has low or high sampling rates. Assuming that the first record corresponds to the start of the trip and the last to its end, and considering the information recorded in the waypoints (date, time tag, longitude, and latitude), then the corresponding trajectory can be associated with a specific departure zone and destination zone for a given day and time.

This zone assignment yields a primary set of OD matrices for each day and time, although these OD matrices correspond to segments of the total population and are strongly biased, since they represent only users of the GPS technology utilized to collect the data. Thus, the sample is not necessarily representative and there are no clear methods for expanding it to the whole population. However, the identified paths in the network can be clustered and analyzed using techniques such as machine learning techniques (Lopez et al., 2017a, 2017b) to identify the paths used and the proportion of their usage for each OD pair. Path choice models like those in (69) can empirically estimate their parameters from the recorded data (Krishnakumari et al., 2019; Ros-Roca et al., 2022). Furthermore, when an equipped vehicle crosses a link and generates a waypoint and correctly map-matches it to the corresponding link, the processed data provides information such as link identity, the time of crossing the link, the origin of the trip, the departure time (time-tag of the first waypoint recorded), and the trip destination. With this information, it is possible to estimate an empirical assignment matrix that allows a reformulation of (80) in order to relate the estimated traffic count to the OD flows:

$$y_{\ell t} = \sum_{(i,j)\in N} \sum_{r=1}^{t} \bar{a}_{ijr}^{\ell t} x_{ijr} \tag{81}$$

where y_{lt} represents the estimated flow in link l at time t; x_{ijr} is the flow departing origin i to destination j at time interval $t \in T$; and $\overline{a}_{ijr}^{lt}$ is the estimated assignment matrix, which is the fraction of trips from origin i to destination j, departing from i at time interval r reaching link l at time t, estimated from the GPS traces. In other words, this data-driven approach reformulates the OD calibration problem by replacing the analytical approaches for estimating the model parameters with empirical estimations from ICT applications. Behara (2019) proposes an alternative approach based on estimating partial path travel times from Bluetooth measurements obtained suitably located antennas in the network.

Nevertheless, all these approaches by Krishnakumari et al. (2019), Mitra et al. (2020), Behara (2019), and Ros-Roca et al. (2022) still complete the model formulation in (79) by optimizing the value of an objective function $\mathcal{F}\left(X, \widehat{X}, Y, \widehat{Y}\right)$ that minimizes a distance measure between the estimated $\widehat{X}$ of the OD matrix and a target OD matrix X, as well as between the estimated link flow counts $\widehat{Y}$ (obtained from (81)) and the observed link flow counts Y (or the estimated and measured partial path travel times).

3.3. Extracting traffic data from image processing

In 2001 the Federal Highway Administration (FHWA) initiated an intense debate about the validity and application of traffic simulation models for traffic analysis. Consequently, in 2002, the FHWA Traffic Analysis Tool Program (https://ops.fhwa.dot.gov/trafficanalysistools/ngsim.htm) was launched to address the questions raised and to establish a methodological framework for the construction and utilization of transport models. The FHWA acknowledged (Alexiadis, Colyar and Halkias, 2007) that microscopic traffic simulators can help evaluate complex scenarios by intricately modeling real-world transportation networks, a task that is challenging using more conventional methods. Moreover, advancements in computer technologies have enabled these simulators to model larger and more complex transportation systems, thereby supporting associated decision-making processes.

From the very beginning, the stakeholders involved in the program unanimously agreed on the premise put forth in this paper: Microscopic models need data, particularly detailed microscopic data that are not easily obtained and not always available. A comprehensive understanding of microscopic traffic flow and car following behavior is crucial for advancing traffic flow theory. This understanding is essential for constructing traffic simulation models, and the most effective means of acquiring such knowledge is by collecting empirical data and providing it as evidence. Consequently, a companion program called the *next generation simulation* (NGSIM) program (https://ops.fhwa.dot.gov/trafficanalysistools/ngsim.htm) was launched with the aim of developing driver behavior algorithms for microscopic modeling by collecting detailed, high-quality traffic datasets. Multiple data collection sites were equipped, and the collected datasets are freely available at the corresponding websites. Notable among them are Interstate I-80 Highway (https://www.fhwa.dot.gov/publications/research/operations/06137/index.cfm) and US Highway 101 (https://www.fhwa.dot.gov/publications/research/operations/07030/index.cfm).

The I-80 Highway location is shown in Figure 25. According to the dataset provided in the website's NGSIM Fact Sheet, "the NGSIM program collected detailed vehicle trajectory data on eastbound I–80 in the San Francisco Bay area in Emeryville, CA, on April 13, 2005. The study area was approximately 500 meters (1,640 feet) in length and consisted of six freeway lanes, including a *high-occupancy vehicle* (HOV) lane. An onramp also was located within the study area. Seven synchronized digital video cameras, mounted from the top of a 30-story building adjacent to the freeway, recorded vehicles passing through the study area. NG-VIDEO, a customized software application developed for the NGSIM program, transcribed the vehicle trajectory data from the video. This vehicle trajectory data provided the precise location of each vehicle within the study area every one-tenth of a second, resulting in detailed lane positions and locations relative to other vehicles. A total of 45 minutes of data are available in the full dataset, segmented into three 15-minute periods: 4:00 p.m. to 4:15 p.m.; 5:00 p.m. to 5:15 p.m.; and 5:15 p.m. to 5:30 p.m. These periods represent the buildup of congestion, or the transition between uncongested and congested conditions, and full congestion during the peak period. In addition to the vehicle trajectory data, the I–80 dataset also contains computer-aided design and geographic information system files, aerial orthorectified photos, freeway loop detector data within and surrounding the study area, raw and processed video, signal timing settings on adjacent arterial roads, traffic sign information and locations, weather data, and aggregate data analysis reports".

Video image processing for traffic analysis remains more of an art than a science. While automated tools can provide an initial approximation, it is no easy task to achieve the level of precision required for extracting sufficiently accurate empirical vehicle trajectories to develop traffic flow models. Because even the best image processing tools cannot overcome the inherent complexities of projection errors, occlusions, shadows, the non-rectilinear shapes of real vehicles, and vehicles with colors similar to the pavement, significant human intervention is still required if traffic flow theory is to advance. Figure 26 depicts the propagation of shockwaves collected from the trajectories at the US101 site.

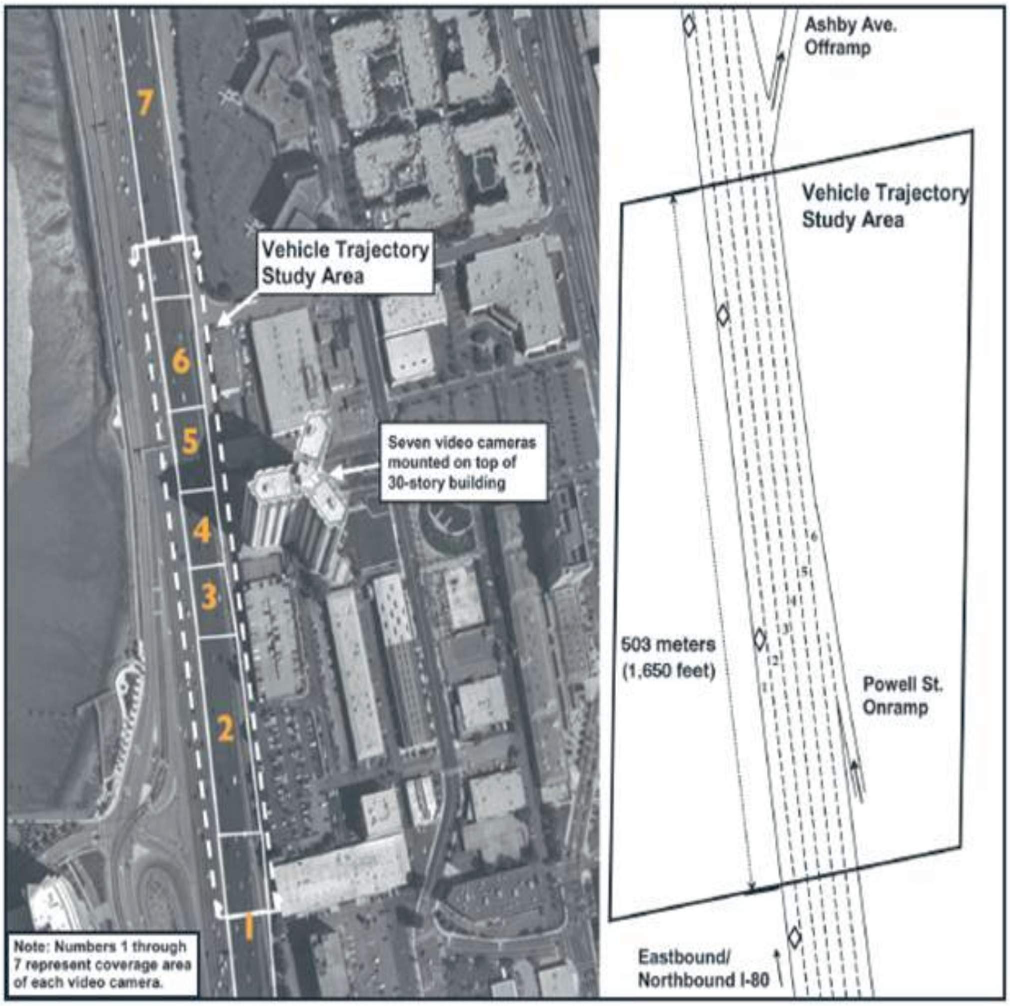

Figure 25. *The aerial photograph on the left shows the extent of the I–80 study area relative to the building where the video cameras were mounted, along with the coverage area for each of the seven video cameras. The schematic drawing on the right shows the number of lanes and location of the Powell Street onramp within the I–80 study area. Source: Public Domain "Federal Highway Administration Research and Technology" https://www.fhwa.dot.gov/publications/research/operations/06137/index.cfm.*

The detailed microscopic data collected by NGSIM were expected to serve as valuable resources for validating traffic simulation models by comparing the values of various vehicle kinematic variables, which include the time and space headways that could be measured, speed distributions, accelerations, and changes in acceleration (jerks). By analyzing these data, we anticipated being able to estimate the parameters of the car-following and lane change models, enabling them to accurately reproduce the observed values.

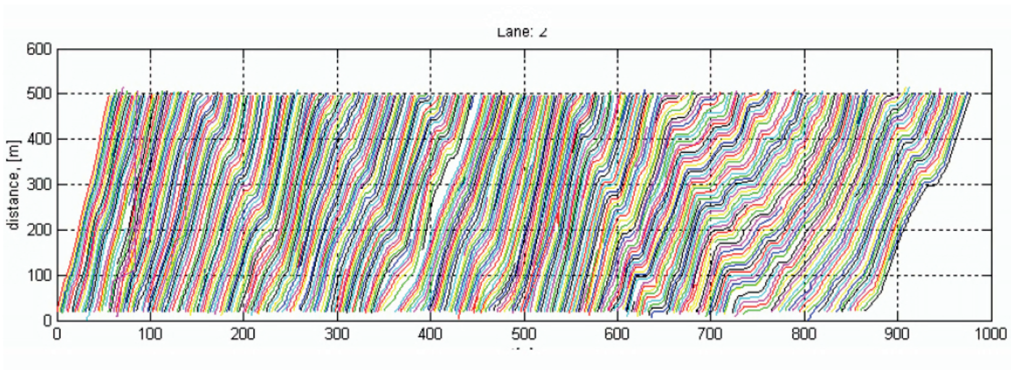

Figure 26. *Shockwave Characteristics (NGSIM, I-80 Dataset, 5:00–5:15 pm, Lane 2).*

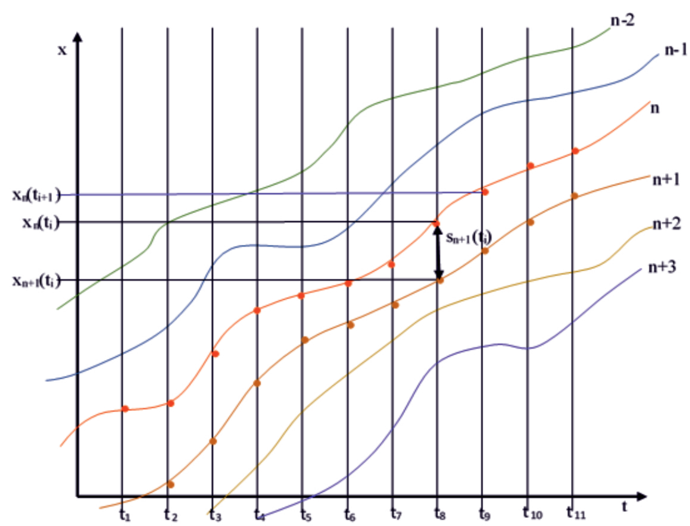

Figure 27. *Hypothetical reconstruction of vehicle trajectories from video processing.*

To summarize the process of extracting information from the trajectories using a hypothetical example, Figure 27 represents a few trajectories obtained through suitable processing of the video data recorded at constant time intervals $\Delta t = t_{i+1} - t_i$ ($\Delta t = 0.1$ seconds for the NGSIM data). The red trajectory corresponds to the *n-th* vehicle, and the red dots along the trajectory indicate the vehicle's positions at each instant t_i, $i = 1, \ldots, T$.

For a given instant t_i, the relative positions of two consecutive vehicles, a leader n and its follower $n+1$, are denoted by $x_n(t_i)$ and $x_{n+1}(t_i)$, respectively. The space headway between them can be calculated as:

$$s_{n+1}(t_i) = x_n(t_i) - x_{n+1}(t_i) \tag{82}$$

For each trajectory, the corresponding time series of speeds can be calculated. For instance, Coifman and Li (2017) propose using the mean difference over multiple time intervals as follows:

$$\widehat{v}_n(t_i) = \frac{x_n(t_{i+p}) - x_n(t_{i-p})}{2p\Delta t} \tag{83}$$

Similarly, accelerations can be derived:

$$\widehat{a}_n(t_i) = \frac{\widehat{v}_n(t_i) - \widehat{v}_n(t_{i-1})}{2\Delta t} \tag{84}$$

Additionally, the jerks (the time change of the acceleration) can be calculated. These and other values can be used to calibrate the parameters of car-following models and test their quality. One early example can be found in Yeo and Skabardonis (2007), who develop, calibrate, and test an improved car-following model based on empirical observations of NGSIM trajectories. Another example is Bevrani and Chung (2011), who modified the Gipps car-following model to enhance its capabilities for safety studies. In the first case, they use the analysis of the trajectories to estimate the probability distribution of space headways estimated from equation (1) and the speed distribution from equation (2). In the second case, the study primarily focuses on the enhanced car-following model's ability to predict the expected time to collision (TTC), a critical indicator of a given traffic situation. TTC for the follower vehicle *n + 1* is calculated as:

$$TTC_{n+1}(t_i) = \frac{x_n(t_i) - x_{n+1}(t_i) - l_n}{\widehat{v}_{n+1}(t_i) - \widehat{v}_n(t_i)} \tag{85}$$

where $\widehat{v}_{n+1}(t_i) > \widehat{v}_n(t_i)$.

The analysis of the I-80 NGSIM data conducted by Yeo and Skabardonis (2007) revealed that the probability distribution of space headways under congested conditions follows a lognormal distribution. The mean of the distribution was found to be 4.24 meters, with a variance of 14.6035 (see Figure 28). A similar distribution was found for the shockwave speeds, and Bevrani and Chung (2011) also found similar lognormal distributions for the TTC.

The theoretical expressions for space headways (82), speeds (83), accelerations (84), and TTC (85), as well as other derived estimates such as jerks or shockwave speeds, are used to estimate empirical values that are later employed for the calibration and validation of car-following models. These expressions implicitly assume that either the empirical values are error-free or their errors have been minimized. However, this assumption is unfortunately not always met. As already discussed in this paper, errors can affect the observed points regardless of the technique used to collect vehicle positions. In the case of the trajectories recorded after processing the video images, these points may be dispersed in the vicinity of the actual physical path followed by the vehicle. These measuring errors can substantially impact the analysis of a follower's and leader's consecutive vehicle behaviors (Punzo, Borzacchiello and Ciuffo, 2011). If $\widehat{f}_n(t)$ is the trajectory function of vehicle n, measurement errors introduce noise into $\widehat{f}_n(t)$,

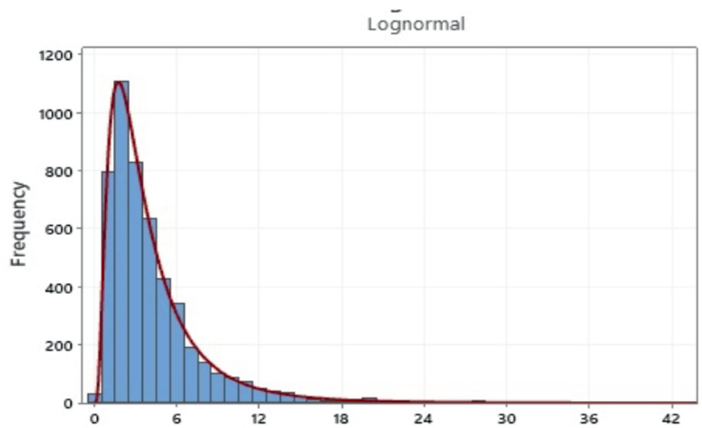

Figure 28. *Lognormal probability distributions of space headways for I80.*

which is magnified by differentiation when calculating speeds, accelerations, and jerks. These quantities represent the physical variations in acceleration over time and can be considered as random components in the traveled space.

Direct use of raw data reveals unacceptable accelerations and physically unreliable jerks. For example, analyzing the acceleration distribution over the entire datasets reveals unfeasible extreme values and anomalous shapes in the distributions, which Punzo et al. (2011) interpret as clear indications of problems in the data collection. They also prove that their analytical evidence shows a positive bias in $\widehat{f_n}(t)$, due to a systematic error component that they believe is inherent to this type of measurement. However, this bias is not self-evident when looking at the trajectory of a single vehicle but becomes apparent when examining the trajectories of consecutive vehicle pairs. Their paper provides analytical evidence of the bias propagated in the vehicle trajectory functions, for which they propose consistency requirements. In a later paper, Montanino and Punzo (2015) refine the procedures for properly reconstructing trajectories, minimizing errors, and making the data useful for the intended purposes. They propose a systematic error analysis based on geometrical and physical considerations, combined with filtering and smoothing techniques. Specifically, they apply Gaussian kernel smoothing to the position data for each trajectory to reduce the impact of noise resulting from data reduction.

Lu and Skabardonis (2007), in a companion paper to Yeo and Skabardonis (2007), also identify the disturbances in the trajectories from NGSIM data due to measurement noise that must be corrected before being used for traffic studies. They apply a Butterworth low pass filter, as proposed by Butterworth (1930).

Many other researchers have identified these limitations of NGSIM datasets and proposed various error filtering and data smoothing techniques to correct them. However, other researchers such as Coifman and Li (2017) claim that "the NGSIM errors are beyond anything that could be corrected strictly through cleaning or interpolation of the reported NGSIM data." In analyzing NGSIM trajectories, they found that their tracking of vehicles quite frequently results in vehicular collisions. Consequently, they systematically remove the NGSIM trajectories to generate a subset of different trajectories that are free of blatant errors. They also employ similar filtering and smoothing techniques, particularly those that use the Savitzky-Golaydigital filter (Savitzky and Golay, 1964). The techniques aim to enhance the precision of the data without distorting its underlying trends, and they involve using a convolutional process that fits successive subsets of datasets in a way similar to a sliding window with a low-degree polynomial. Other authors employ spline fitting methods, resulting in a new dataset that Coifman (2017) has made publicly available for research purposes. He asserts that these cleaned NGSIM data can now serve as a benchmark for assessing the quality of trajectory data.

4. Concluding remarks and insights into some current trends

The main thesis held in this paper is that a proper understanding of a complex system can be achieved by acquiring adequate knowledge about the system and translating it into a modeling hypothesis. The hypothesis should serve as the foundation for explaining how the system works and formally representing it through a model. Models, therefore, become a scientific tool for better understanding a system and supporting rational decisions by providing insights into how the system will behave under other conditions. In other words, models provide answers to what-if questions about the system. As emphasized in Section 1, the system, the observer, and the model form a unit known as the Minsky triad, in which questions are asked about the system and its objectives in order to support rational decision-making. Thus, no unique model of a system exists but instead multiple models that depend on the specific questions asked by the observer. This general modeling theory, outlined in Section 1, applies to various types of systems and specifically to transport systems, which is the focus of this paper.

Transport systems belong to a family of complex systems that can be analyzed using the Minsky triad. Section 2 provides brief examples of the three main families of transport models based on the modeler's perspective regarding the questions that the models aim to answer and the corresponding modeling hypotheses that align with the characteristics of the system that are relevant for answering these questions. Each modeling approach, whether macro (static or dynamic), micro, or meso, is summarized in a subsection that describes the underlying modeling hypotheses and how they are translated into a mathematical formal representation. The models resulting from each approach identify the parameters on which they depend, and the numerical values these parameters must be estimated based on the data.

Another key thesis of this paper holds that data required by models are not in themselves information but instead carry information that requires specific processing. This establishes a two-way interaction between models and data. Models need data, and data can provide only limited useful information without the aid of models. Models are essential for bridging the gap between descriptive and predictive capabilities, as they provide an understanding of the system.

For models to be truly valuable, calibration and validation are necessary. This entails ensuring that the model's parameter values are accurate, thereby establishing the validity of the model and its ability to faithfully reproduce the system's behavior. Section 2 concludes by providing an overview of the calibration and validation processes, emphasizing the pivotal role of data in these processes.

Considering the significant role of data in modeling approaches and the fact that data alone do not inherently reveal their embedded information, it becomes imperative to address critical issues pertaining to data availability, its characteristics based on the employed data collection technology, and the appropriate data processing techniques required to extract the relevant information about each phenomenon generated by this data. This information is crucial for parameter estimation during the calibration process, as well as for comparison during the validation process, ultimately enabling the utilization of the model to answer what-if questions.

Section 3 addresses these topics be establishing a methodological framework for data processing. It provides a general overview of the various types of data and their characteristics, depending on the available technologies. The section also illustrates the use of this methodological framework with a few selected examples based on some of the most recent data collection technologies, namely those supported by ICT applications like Bluetooth, CDR and GPS from mobile devices, and video image processing.

However, this section begins by emphasizing that datasets, regardless of the technology used, always contain errors, missing data, and other flaws that need to be corrected and completed to ensure data completeness and consistency. This is achieved through the application of filtering techniques, one example of which is the powerful Kalman filter, which measures travel times by tagging two consecutive antennas used by mobile Bluetooth devices. The Kalman filter not only identifies and removes outliers, but it also replaces them with the most likely values to obtain a complete and consistent dataset.

The subsequent steps demonstrate the utilization of data provided by two prevalent ICT applications to generate dynamic OD matrices, which serve as crucial inputs for microscopic and mesoscopic traffic models. Dynamic OD matrices reveal the time-dependent traffic patterns, which can be identified by techniques that either track the CDRs of mobile phones associated with phone cells corresponding to the antennas along their paths or record the GPS waypoints that track the trajectory of mobile devices. In both cases, ad hoc filtering procedures are necessary to remove erroneous records or ensure the validity of the records. This involves ensuring correct matching between CDR and geographic coverage of TAZs and phone cells for mobile phones, as well as map matching between waypoints and their physical locations on the road network. These specific filtering processes are briefly discussed and illustrated. However, the OD estimates in both cases have limitations. They may either provide global estimates of trips without distinguishing the mode of transport used, or they only correspond to a specific mode, such as cars, for which only a subsample is available (i.e., equipped vehicles). Consequently, additional information from other sources is required in both scenarios, either to split the OD into the various transport modes or to find a way to extend the sample to the whole population. The most commonly used sources are prior information from conventional surveys in the form of target OD matrices and link flow counts from conventional detection stations that serve as reference ground truth. This section of the paper indicates how specific optimization techniques can be used to achieve the objectives.

Finally, Section 3 concludes with a representative example of using video image processing to extract traffic information, specifically from the FHWA's NGSIM project, aimed at providing traffic datasets for testing traffic simulation models, particularly car-following models that are fundamental to microscopic simulation engines. The section describes the datasets, their processing, and the filtering techniques employed, while also highlighting the controversy surrounding the datasets since their inception. This example effectively demonstrates the advantages and disadvantages associated with certain uses of technologies in extracting valid data and how these challenges have been overcome.

We could conclude here, as these remarks have highlighted how the thesis stated at the beginning of the paper has been demonstrated through significant examples. By identifying the hypotheses underlying the key transport models and illustrating the interdependence between models and data, it is evident that each relies on the other and neither can replace or render the other unnecessary. However, ending at this point may leave a sense of incompleteness. It is important to provide a glimpse of current trends and what lies ahead.

Numerous avenues of exploration can be identified, but two dominant themes emerge, considering that models used to analyze transport systems are also tools for analyzing the mobility they facilitate. These themes seek to offer insights into the question: What factors can determine the urban mobility of the future?

4.1. Scenarios dominated by technological developments: the case of connected and autonomous vehicles

For those who believe that the future of mobility will be fundamentally determined by technology such as connected and autonomous vehicles (CAV), electric vehicles, and information and communication technologies (ICT), understanding the future of mobility requires models that take into account the influence of these technologies. This perspective implicitly assumes that technology will enable people to travel from origins to destinations for the same reasons as today while selecting the most convenient paths, but with the advantage of CAVs making choices based on data collected from other CAVs in addition to conventional information. The key modeling challenge then becomes how car-following models will function, not only for pairs of vehicles but also for groups or platoons of interconnected vehicles traveling in a coordinated manner. It is crucial to determine the conditions under which the dynamics of the platoon will remain stable. To provide a comprehensive overview, it is worth mentioning a seminal work inspired by the car-following approaches discussed in Section 2. Building upon the general modeling approach proposed by Ward and Wilson (2011) and Wilson (2011), which formulates car-following models in terms of a functional framework modeling the follower's reaction in terms of acceleration or deceleration based on speeds, spacings, and relative speeds:

$$a_{n+1}(t) = \ddot{x}_{n+1}(t) = \mathcal{F}\left[s_{n+1}(t), \Delta v_{n+1}(t), v_{n+1}(t)\right] \tag{57}$$

These models have "uniform flow" steady solutions (equilibria) if, for each $s^* > l$, there is a $v^* = V(s^*) > 0$ such that $\mathcal{F}(s^*, 0, v^*) = 0$, where $V(s^*)$ is the equilibrium speed-spacing relationship that leads to a fundamental diagram. Researchers such as Wagner (2016) and Talebpour and Mahmassani (2016) deem this functional framework to be a suitable starting point for studying the behavior of autonomous vehicles, due to its generic approach that does not assume specific driver characteristics and can therefore capture interactions among autonomous vehicles with nonhuman drivers.

Ward and Wilson (2011) define the string stability of a platform in terms of the response to a leader suddenly braking and the decaying perturbation as it propagates upstream within the platoon. In this case, the car-following model is considered platoon stable. Then the analytical conditions string stability can be expressed in terms of the partial derivatives $\mathcal{F}_s$, $\mathcal{F}_{\Delta v}$, $\mathcal{F}_v$ of the functional $\mathcal{F}[s(t), \Delta v(t), v(t)]$, evaluated at $(s^*, 0, v(s^*))$, as follows:

$$\begin{aligned} \mathcal{F}_s^n &= \left.\frac{\partial \mathcal{F}(s_n, \Delta v_n, v_n)}{\partial s_n}\right|_{(s^*,0,v(s^*))} \\ \mathcal{F}_{\Delta v}^n &= \left.\frac{\partial \mathcal{F}(s_n, \Delta v_n, v_n)}{\partial \Delta v_n}\right|_{(s^*,0,v(s^*))} \\ \mathcal{F}_v^n &= \left.\frac{\partial \mathcal{F}(s_n, \Delta v_n, v_n)}{\partial v_n}\right|_{(s^*,0,v(s^*))} \end{aligned} \tag{85}$$

and:

$$\sum_n \left[\frac{\mathcal{F}_v^{n2}}{2} - \mathcal{F}_{\Delta v}^n \mathcal{F}_v^n - \mathcal{F}_s^n\right]\left[\prod_{m=n} \mathcal{F}_s^m\right]^2 \tag{86}$$

where the index n covers the set of vehicles in the platoon. In the case of Talebpour and Mahmassani (2016), string stability is evaluated in terms of the intelligent driver model (IDM) developed Kesting et al. (2010) and defined by equations (55) and (56), where each vehicle in the platoon will have specific model parameters associated with it.

Considering that the autonomous vehicles are equipped with monitoring capabilities for all vehicles in their vicinity, their time lags and anticipation times can be estimated in terms of sensing and mechanical delays. The speeds of autonomous vehicles in the platoon should allow them to come to a full stop when the leader initiates maximum deceleration by braking.

They analyze the string stability of the proposed model following the approaches of Ward (2009), Ward and Wilson (2011), and Treiber and Kesting (2013) for a homogeneous platoon of vehicles. The partial derivatives (86) are calculated to evaluate it in terms of equation (87).

$$
\begin{aligned}
\mathcal{F}_s &= \frac{2a}{s^*}\left(\frac{s_0 + T v\left(s^*\right)}{s^*}\right)^2 \\
\mathcal{F}_{\Delta v} &= -\frac{v\left(s^*\right)}{s^*}\sqrt{\frac{a}{b}}\left(\frac{s_0 + T v\left(s^*\right)}{s^*}\right) \\
\mathcal{F}_v &= -\frac{a\delta}{v_0}\left(\frac{v}{v_0}\right)^{\delta-1} - \left(\frac{2aT}{s^*}\right)\left(\frac{s_0 + T v\left(s^*\right)}{s^*}\right)
\end{aligned}
\tag{87}
$$

The resulting partial derivatives are expressed as functions of the vehicle speed v, the equilibrium gap s^*, and the equilibrium speed $v\left(s^*\right)$. These expressions can be simplified using the equilibrium relationships proposed by Treiber and Kesting (2013):

$$
s^*(v) = \frac{s_0 + vT}{\sqrt{1 - \left(\frac{v}{v_0}\right)^{\delta}}} \tag{88}
$$

From this, Talebpour and Mahmassani (2016) conduct an analysis of stable and unstable scenarios based on the parameter values governing the model, taking into account the suggested values from empirical evidence of cruise control studies. This example is selected to be consistent with the models discussed in Section 2, which can only be studied analytically or through simulation, since the real systems are not yet implemented. It illustrates how models can assist in the design and testing of new systems. Furthermore, considering that time-lags can mainly depend on sensing delays, which are strongly influenced by telecommunication technologies, modeling approaches that also include telecommunication aspects have been explored. One early example is Talebpour, Mahmassani and Bustamante (2016), and a more recent one is Dai et al. (2022). Let us close these comments by mentioning other types of modeling approaches to car following models, such as those inspired by reinforced learning processes (Wu et al., 2017). So far, these approaches can only be tested through simulations due to the lack of observed data.

4.2. Scenarios dominated by other factors: urban forms, accessibility, etc.

For those who acknowledge the influence of non-technological factors, such as urban forms and their impact on the temporal and spatial distribution of activities, the future of mobility is intertwined with the evolution of cities and the complex relationships between mobility, urban forms, and transport systems. This perspective assumes that technology enables new possibilities like telecommuting, which eliminates the need for physical displacement to overcome physical distances, or the concept of the ”15-minute city,” where urban areas are designed to reduce the necessity of extensive travel by prioritizing non-motorized modes of transportation over motorized ones. Consequently, alternative models are required to explore these aspects. What are these models?

We need to shift our mindset, as suggested by Barceló (2019), from a conventional reductionist approach to a complex dynamic systems approach. In this perspective, a complex system is a system composed of a large number of interacting components that, as a whole, exhibit properties that are distinct from the properties of the individual components. This implies taking a holistic view of the whole as different from the sum of the parts. Thus, the transport system comprises various interconnected networks for different modes of transportation, such as cars, buses, metro, and railways, which need to be integrated to accurately capture their interactions. This paradigm shift challenges the traditional modeling approach focused solely on trips and their purposes, and instead seeks to understand the underlying causes and consequences. Central to this perspective are the activities that drive mobility, including economic, leisure, and shopping activities. Accessibility to these activities becomes the key factor in explaining the need for people and goods to travel, bridging the spatial separation of activities resulting from the urban spatial structure determined by land use. The transport infrastructure plays a crucial role in providing the physical connectivity required to bring people to their desired activities.

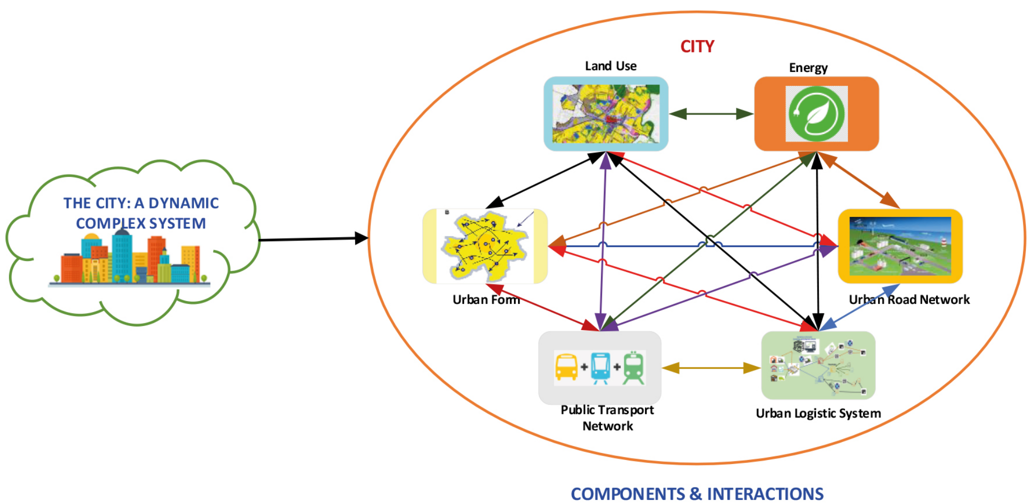

Figure 29. *The complex systems and components of the city.*

However, a holistic perspective cannot overlook the fact that the transport system is reliant on energy, particularly when considering sustainability and the associated energy consumption and emissions from transportation. Shifting towards more sustainable transport technologies, such as replacing fossil fuel-powered vehicles with electric vehicles, necessitates the inclusion of energy grids as part of the system. Figure 29 schematically depicts this approach, presenting the city as a complex system comprising key components: urban form, land use, energy, road network, public transport networks, and the logistics system, which is responsible for ensuring goods reach activity locations but is often overlooked in conventional approaches. The interactions of these components shape the functioning of the overall system.

This paradigm shift changes the standard modeling approaches and calls for the transition from individual transport system models (e.g., private, public, urban logistics) to a comprehensive model of the city that incorporates urban form, land use, integrated transport networks, energy systems, and charging grids. In recent years, various modeling tools have been developed to support this new approach. One such tool is UrbanSim, an open-source platform designed by Waddell (2011), Waddell (2015), and Waddell et al. (2018). It can integrate with transport planning modeling software like SATURN (Hall, Van Vliet and Willumsen, 1980) and Visum (PTV AG, 2020). Transport modeling has progressed beyond the basic four-step model described in Section 2, which assumes that trips originating from one TAZ and destined for other TAZs are solely determined by the socioeconomic characteristics of the TAZs, implicitly depending on land use. Consequently, changing land use characteristics will also change the number of trips generated in the TAZ. Land use transport integrated (LUTI) models developed by Wegener and Fürst (1999), Acheampong and Silva (2015), and van Wee (2015) explicitly account for these interdependencies. The integration of transport planning software into UrbanSim represents a notable advance in this modeling direction, and early examples can be found in reports from the EU project SIMBAD (Nicolas and Zuccarello, 2011; Dasigi, 2015).

Figure 29, adapted from SIMBAD Project, depicts the conceptual diagram and the flow of information and data between the various modules in this integrated framework, which also includes an urban freight model that addresses the previously overlooked freight traffic flows in conventional models.

A more recent and more powerful software platform for city modeling that integrates ad hoc models for each component is SimMobility (Adnan et al., 2016; Zhu et al., 2018). This software is described on the MIT SimMobility website (https://mfc.mit.edu/simmobility) as follows: “SimMoblity is the simulation platform of the Future Urban Mobility Research Group at the Singapore-MIT Alliance for Research and Technology (SMART) that aims to serve as the nexus of Future Mobility research evaluations. It integrates various mobility-sensitive behavioral models with state-of-the-art scalable simulators to predict the impact of mobility demands on transportation networks, services, and vehicular emissions.

intelligent transportation The platform enables the simulation of the effects of a portfolio of technology, policy, and investment options under alternative future scenarios. Specifically, SimMobility encompasses the modeling of millions of agents, from pedestrians to drivers, from phones and traffic lights to GPS, from cars to buses and trains, from second-by-second to year-by-year simulations, across entire countries”. As this presentation highlights, SimMobility offers the additional advantage of being an activity-based approach that fully integrates urban freight transport (Sakaia et al., 2020).

Upon analyzing the dynamics of transportation, it becomes apparent that it is significantly more intricate than the typical simplifications employed in four-step trip-based models. In these models, the trip serves as the fundamental unit of analysis, treating in-

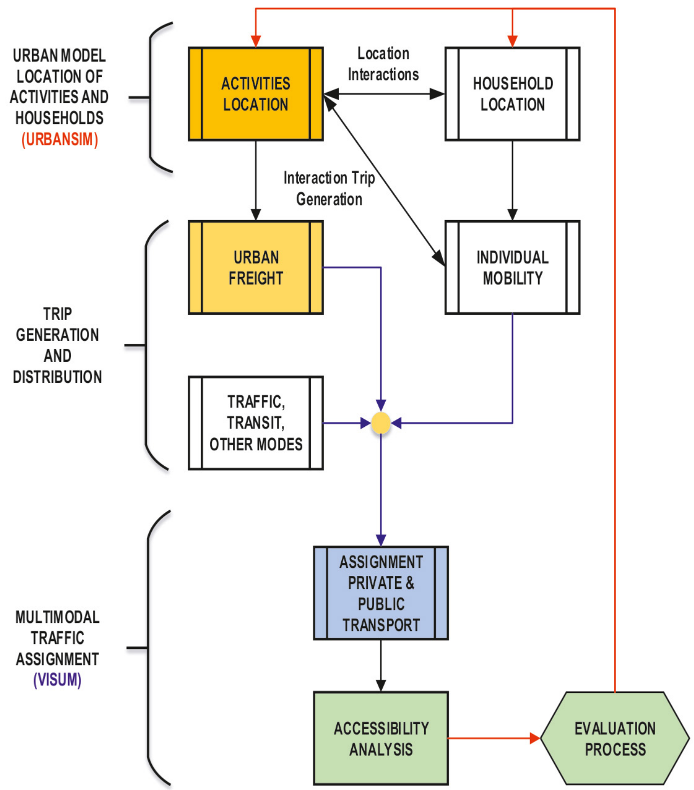

Figure 30. *Conceptual diagram of integrating UrbanSim with the Visum transport model and an urban freight model.*

dividual trips as independent and separate entities. However, when considering the daily schedules of individuals and their activities, we must look at them in terms of sequential chains, like the one illustrated in Figure 31, where a sequence is defined as a series of time points where a person transitions from one discrete state (activity) to another.

In the generic example in Figure 31, the person starts at origin O (home, for example) and travels to activity A_i, perhaps taking their children to school by walking. The duration of this activity is t_i. The person then travels for a duration of τ_{ij} by a transport mode such as a bus from the location of activity A_i to the location of activity A_j, say, to work during duration t_j. He or she then travels by a transport mode that may be the same as or different from the location of activity A_j to the location of activity A_k (say, shopping) over a duration that is τ_{jk} time units. After t_k time units, activity A_k is completed and the individual moves to another destination, D.

From this description, it is clear that the conventional four-step trip-based models lack the necessary structure to represent either a journey's sequential decisions, which now appear as an intermodal chain, or their interrelationships.

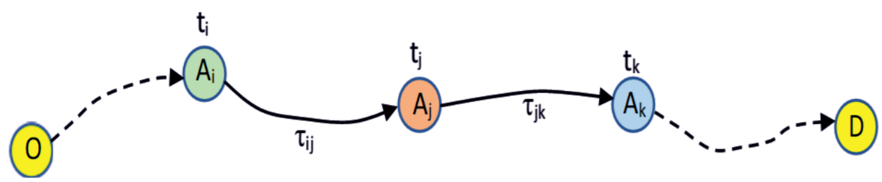

Figure 31. *Examples of activity sequences in a daily schedule.*

The new approach, exemplified by models such as SimMobility and MATSim (Horni, Nagel and Axhausen, 2016), focus on replicating actual traveler decisions by thoroughly understanding the motivations and processes behind them. These models aim to comprehensively represent various interrelated aspects, such as the types of activities individuals engage in, the locations and timing of these activities, and the modes of transportation used to reach them. This entails not only the ability to generate and schedule the activities, which provides insights into the activities people participate in, but also to generate tours and trips with specific destinations and modal choices for reaching them. By considering these factors, the models can identify the routes and modes individuals will utilize, leading to the subsequent network assignment.

Generating the schedule of activities, as depicted in Figure 31, consists of identifying the number and type of activities, their sequential order, the start time and duration of each activity, the modal choices, and the routes taken.

This analysis is conducted through an agent-based simulation, in which individuals are represented as agents whose behaviors are modeled by the decision processes generated through an activity-based approach. Agent-based simulation explicitly incorporates multimodality by simulating the available transportation modes such as cars, buses, and metros, allowing agents to switch between modes according to their schedules.

Additionally, agent-based simulation can effectively address urban freight transport by considering fleets of vehicles and agents in order to schedule their activities as sequences of visits for pickups and deliveries.

Figure 32 depicts the logical diagram of agent-based simulation supported by the activity-based approach. In terms of structural components, it shares similarities with the assignment, mesoscopic, and microscopic models discussed in Section 2. This is because the network supply model, which includes the networks of all available transportation modes, must be built using the same data sources (i.e., GIS and all complementary urban information) that are typically used in transport modeling.

Activity-based models require a huge amount of data, since they must generate information by combining socioeconomic (census tracts) and land-use data with survey data by employing specific sampling techniques like Gibbs sampling to generate synthetic populations, which will in turn be used to generate agents and their activity plans. A seminal work on these applications to agent-based simulation can be found in Farooq et al. (2013), and a more comprehensive overview of available methods can be found

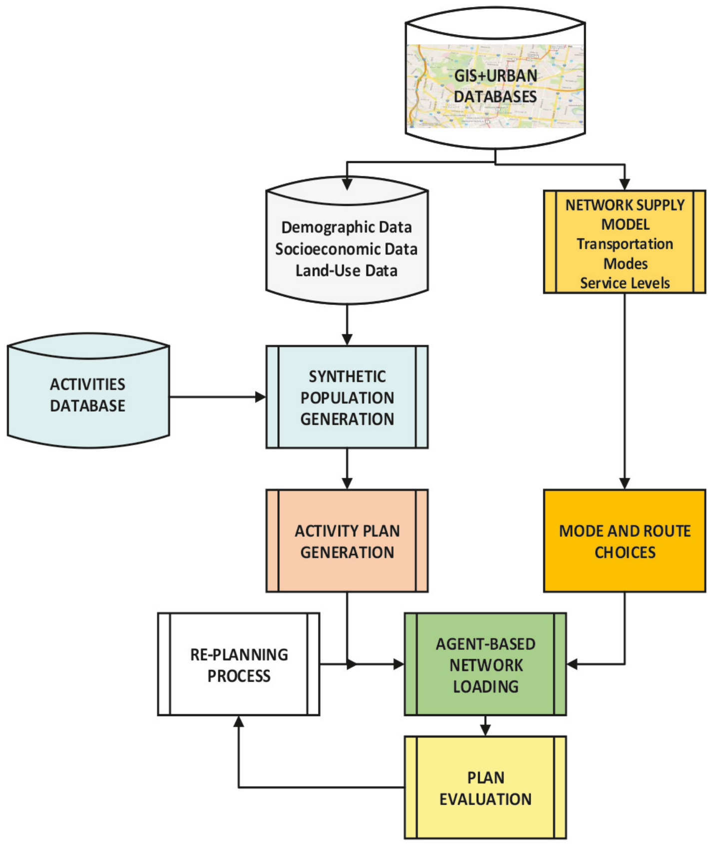

Figure 32. *Activity- and agent-based simulation models.*

in Chapuis, Taillandier and Drogoul (2022). One example of applying MATSim using available data for Barcelona, which was previously discussed in Section 3.2.1 regarding the use of mobile phone data, can be seen in the work of Bassolas et al. (2019). The logic diagram in Figure 30 describes the simulation process as an iterative process, where the performance is evaluated using suitable indicators. Models like SimMobility and MATSim provide sets of indicators, and changes are introduced accordingly, such as adjustments in route or modal choices based on discrete choice models, thereby aiming to achieve some form of equilibrium while emulating individual behaviors.

Funding: This research project has been funded by Spanish R+D Programs, specifically under Grant PID2020-112967GB-C31.

References

Acheampong, R.A. and Silva, E. A. (2015). Land use–transport interaction modeling: A review of the literature and future research directions. *The Journal of Transport and Land Use*, 8(3), 11-38. http://jtlu.org

Ackoff, R.L., Gupta, S.K. and Minas, J.S. (1965). *The Scientific Method. Optimizing Applied Research Decisions*. John Wiley.

Adnan, M., Pereira F.C., Azevedo, C.M.L., Basak, K., Lovric, M. Feliu, S.R., Zhu, Y., Ferreira, J., Zegras, C. and Ben-Akiva, M.E. (2016). *SimMobility: A Multi-Scale Integrated Agent-based Simulation Platform*. 95th 42 Annual Meeting, Transportation Research Board.

Ahmed, K.I. (1999). *Modeling Drivers' Acceleration and Lane Changing Behaviors*. PhD thesis, Massachusetts Institute of Technology.

Akcelik, R. (1991). Travel time functions for transport planning purposes: Davidson's function, its time-dependent form and an alternative travel time function. *Australian Road Research*, 21, 49-59.

Alexander, L., Jiang, S., Murga M. and González, M.C. (2015). Origin–destination trips by purpose and time of day inferred from mobile phone data. *Transportation Research Part C*, 58, 240-250 http://dx.doi.org/10.1016/j.trc.2015.02.018.

Alexiadis, V., Colyar, J. and Halkias, J. (2007). A Model Endeavor, *Public Roads*, 70(4). Publication Number: FHWA-HRT-07-002.

Anderson, C. (2008). *The End of Theory: The Data Deluge Makes the Scientific Method Obsolete*. wired.com.

Antoniou, C., Barceló, J., Breen, M., Bullejos, M., Casas, J., Cipriani, E., Ciuffo, B., Djukic, T., Hoogendoorn, S. P., Marzano, V. et al. (2016). Towards a Generic Benchmarking Platform for Origin-Destination Flows Estimation/Updating Algorithms: Design, Demonstration and Validation. *Transp. Res. Part C Emerg. Technol.*, 66, 79-98. https://doi.org/10.1016/j.trc.2015.08.009.

Asakura, Y. and Hato, E. (2009). Tracking Individual Travel Behavior using Mobile Phones: recent technological developments. Chapter 9 in:The Expanding Sphere of Travel Behavior Research. *Selected papers from the 11th International Conference of Travel Behavior Research. Emerald Publishing Limited*. ISBN: 978-1- 84855-936-3.

Asakura, Y. and Hato, E. (2004). Tracking survey for individual travel behaviour using mobile communication instruments. *Transportation Research Part C*, 12, 273-291.

Bando, M., Hasebe, K., Nakanishi, K. and Nakayama, A. (1998). Analysis of optimal velocity model with explicit delay. *Physical Review E*, 58, 5.

Bando, M., Hasebe, K., Nakayama, A., Shibata, A. and Sugiyama, Y. (1995). Dynamic model of traffic congestion and numerical simulation. *Phys. Rev. E*, 51, 1035-1042.

Barceló, J., Ros-Roca, X. and Montero, L. (2022). Data Analytics and Models for Understanding and Predicting Travel Patterns in Urban Scenarios. Chapter 7 in *The Evolution of Travel Time Information Systems*, M. Martínez-Díaz (ed.), Springer Tracts on Transportation and Traffic 19. Doi: 10.1007/978-3-030-89672- 0_6.

Barceló, J. (2019). Future trends in sustainable transportation, Chapter 16 in *Sustainable Transportation and Smart Logistics. Decision Making Models and Solutions*, Editors: J. Faulin, S. Grasman, A. Juan, P. Hirsch, Elsevier, ISBN: 978-0-12-814242-4.

Barceló, J. (2015). Analytics and the Art of Modeling, *International Transactions in Operations Research (ITOR)*, 22, 429-471.

Barceló, J., Gilliéron, F., Linares, M.P., Serch, O. and Montero, L. (2012). Exploring Link Covering and Node Covering Formulations of Detection Layout Problem. *Transportation Research Records: Journal of the Transportation Research Board*, 2308, 17-26.

Barceló, J. (2010). Models, Traffic Models, Simulation and Traffic Simulation, Chapter 1 in J. Barceló, Ed. *Fundamentals of Traffic Simulation*, Springer, ISBN: 978-1-4419-6142-6.

Barceló, J., Montero, L., Marqués, L. and Carmona, C. (2010). Travel time forecasting and dynamic of estimation in freeways based on Bluetooth traffic monitoring, *Transportation Research Records: Journal of the Transportation Research Board*, 2175, 19-27.

Bar-Gera, H. (2002). Origin-based algorithms for the traffic assignment problem. *Transportation Science*, 36, 398- 417.

Bassolas, A., Ramasco, J.J., Herranz, R. and Cantu-Ros, O.G. (2019). Mobile phone records to feed activity-based travel demand models: MATSim for studying a cordon toll policy in Barcelona. *Transportation Research Part A*, 121 56-74, https://doi.org/10.1016/j.tra.2018.12.024

Behara, K.N.S. (2019). Origin-Destination Matrix Estimation Using Big Traffic Data: A Structural Perspective. *School of Civil Engineering and Built Environment Science and Engineering Faculty Queensland University of Technology*.

Ben-Akiva, M., Koutsopoulos, H.N., Antoniou, C. and Balakrishna, R. (2010). *Traffic Simulation with DynaMIT*. In *Fundamentals of Traffic Simulation*, edited by Barceló, J. Switzerland: Springer, ISBN 978-1-4419-6142-6.

Ben-Akiva, M., Bierlaire, M., Burton, D., Koutsopoulos, H.N. and Mishalani, R. (2001). Network State Estimation and Prediction for Real-Time Traffic Management. *Networks and Spatial Economics*, 1, 293-318.

Ben-Akiva, M. and Bierlaire, M. (1999). Discrete Choice Methods and Their Applications to Short Term Travel Decisions. In Hall, R. W. (ed.). *Handbook of Transportation Science*. http://roso.epfl.ch/mbi/handbook-final.pdf

Ben-Akiva, M. and Lerman, S. (1985). Discrete Choice Analysis: Theory and Application to Travel Demand. *Transportation Studies*. Massachusetts: MIT Press.

Bessa, J., de Magalhães V. and Santos, G.H. (2021). Calibration and Validation of a Volume-Delay Function using Genetic Algorithms. *Journal of Urban and Environmental Engineering*, 15(3), 173-179. DOI:10.4090/juee. 2021.v15n2.173179.

Bevrani, K. and Chung, E. (2011). Car following model improvement for traffic safety metrics reproduction. In *Proceedings of the Australasian Transport Research Forum 2011*, PATREC, Adelaide Hilton Hotel, Adelaid, SA, 1-14.

Blackett, P.M.S. (1948). Operational Research. *The Advancement of Science*, 5(17), 26-38.

Bovy, P., Bekhor, S. and Prato, C. (2008). The factor of revisited path size. *Transp. Res. Board*, 2076, 132-140. http://dx.doi.org/10.3141/2076-15.

Box, G.E.P. and Draper, N.R. (1987). *Empirical Model-Building and Response Surfaces*, John Wiley & Sons.

Box, G.E.P. (1976). *Journal of the American Statistical Association*, 71(356), 791-799.

Bunge, M. (1960). *La ciencia, su método y su filosofía*. Buenos Aires : Eudeba. (In French : *La science, sa méthode et sa philosophie*. Paris: Vigdor, 2001. [ISBN 2-910243-90-7]).

Bureau of Public Roads (1964). *Traffic Assignment Manual*. Urban Planning Division, US Department of Commerce, Washington, DC.

Butterworth, S.(1930). On the Theory of Filter Amplifiers. *Experimental Wireless and the Wireless Engineer.*, 7, 536- 541. https://www.changpuak.ch/electronics/downloads/On_the_Theory_of_Filter_Amplifiers.pdf

Cascetta, E. (2001). *Transportation System Engineering: Theory and Methods*. Kluwer, ISBN: 0-7293-6972-8.

Cascetta, E., Nuzzolo, A., Russo, F. and Vitetta, A. (1996). A modified logit route choice model overcoming path overlapping problems. In: *Proceedings of the 13th International Symposium on the Theory of Road Traffic Flow*.

Castillo, E., Conejo, A.J., Menéndez, J.M. and Jiménez, P. (2008). The Observability Problem in Traffic Network Models. *Computer-Aided Civil and Infrastructure Engineering*, 23, 208-222.

Chandler, R.E., Herman, R. and Montroll, E.W. (1958). Traffic Dynamics: Studies in Car Following. *Opns. Res.*, 6, 165-184.

Chapuis, K., Taillandier, P. and Drogoul, A.(2022). Generation of Synthetic Populations in Social Simulations: A Review of Methods and Practices. *Journal of Artificial Societies and Social Simulation*, 25(2), 6. Doi: 10.18564/jasss.4762 Url: http://jasss.soc.surrey.ac.uk/25/2/6.html

Chiu, Y.C., Bottom, J., Mahut, M., Paz, A., Balakrishna, R., Waller, T. and Hicks, J. (2011). Dynamic traffic assignment: a primer. *Transportation Research E-Circular*, (E-C153).

Cluet, J. (2021). Analysis of GPS tracking data for traffic modelling parameters estimation. Bachelor Thesis report. *Computer Science Degree (GEI)*, Facultat d'InformÃ tica de Barcelona, Universitat PolitÃ¨cnica de Catalunya.

Codina, E., Ibáñez, G. and Barceló, J. (2015). Applying Projection-based Methods to the Asymmetric Traffic Assignment Problem. *Computer-Aided Civil and Infrastructure Engineering*, 20(2), 103-119.

Coifman, B. (2017). *Data Sets*. https://www.ece.osu.edu/?coifman/documents.

Coifman, B. and Li, L. (2017). A critical evaluation of the Next Generation Simulation (NGSIM) vehicle trajectory dataset. *Transportation Research Part B*, 105, 362-377.

Çolak, S., Lima, A. and González, M.C. (2016). Understanding congested travel in urban areas. *Nature Communications*. DOI: 10.1038/ncomms10793

Daganzo, C.F. (1997). *Fundamentals of transportation and traffic operations*. Pergamon Oxford.

Daganzo, C.F. (1994). The cell-transmission model: a simple dynamic representation of highway traffic. *Transportation Research Part B: Methodological*, 28(4), 269–287.

Daganzo, C.F. (1995a). The cell transmission model part II: network traffic. *Transportation Research Part B: Methodological*, 29, 79-93.

Daganzo, C.F. (1995b). A finite difference approximation of the kinematic wave model of traffic flow. *Transportation Research Part B: Methodological*, 29(4), 261-276.

Dai, Y., Yang, Y., Zhong, H., Zuo, H. and Zhang, Q. (2022). Stability and Safety of Cooperative Adaptive Cruise Control Vehicular Platoon under Diverse Information Flow Topologies. *Hindawi, Wireless Communications and Mobile Computing*, Volume 2022, Article ID 4534692, https://doi.org/10.1155/2022/4534692

Dailey, D.H., Harn, P. and Lin, P-J. (1996). Final Research Report, Research Project T9903, Task 9, ATIS/ATMS Regional IVHS Demonstration, prepared for *Washington State Transportation Commission*, Department of Transportation.

Dasigi, S. (2015). *An integrated approach linking Land Use and socioeconomic characteristics for improving travel demand forecasting*. University of Cincinnati.

de Montjoye, Y-A., Smoreda, Z., Trinquart, R., Ziemlicki, C. and Blondel, V. D. (2014). *D4D-Senegal: The Second Mobile Phone Data for Development Challenge*, http://arxiv.org/pdf/1407.4885v2.

Dial, R. (2006). A path-based user equilibrium traffic assignment algorithm that obviates paths storage and enumeration. *Transportation Research Part B*, 40, 917-936.

Djukic, T. (2014). Dynamic OD Demand Estimation and Prediction for Dynamic Traffic Management, TU Delft, 2014. https://doi.org/10.4233/uuid:ab12d7a7-e77b-424d-b478-d58657f94dd1

Edie, L. (1963). Discussion of traffic stream measurements and definitions, in: Almond, J. (Ed.). *Proceedings of the 2nd International Symposium on the Theory of Traffic Flow*, 139-154.

Farooq, B., Bierlaire, M., Hurtubia, R. and Flötteröd, G. (2013). Simulation based population synthesis. *Transportation Research Part B: Methodological*, 58, 243-263.

Florian, M. and Constantin, I. (2009). A new look at the projected gradient method for equilibrium assignment. *Transportation Research record Journal of the Transportation research Board*. Paer 2090, pp. 10-16. DOI:10.3141/2090-02.

Florian, M., Mahut, M. and Tremblay, N. (2001). A Hybrid Optimization-Mesoscopic Simulation Dynamic Traffic Assignment Model. *Proceedings of the 2001 IEEE Intelligent Transport Systems Conference*, 2001, Oakland, 118-123.

Florian, M., Mahut, M. and Tremblay, N. (2002). Application of a Simulation-Based Dynamic Traffic Assignment Model. *International Symposium on Transport Simulation*, 2002, Yokohama. (also in: *Simulation Approaches in Transportation Analysis*, 2005, edited by R. Kitamura and M. Kuwahara, US: Kluwer.)

Florian, M. and Hearn, D. (1995). Network Equilibrium Models and Algorithms, Chapter 6 in: M.O. Ball et al., Eds., *Handbooks in OR and MS*, Vol.8, Elsevier Science B.V.

Florian, M., Guelat, J. and Spiess, H. (1987). An Efficient Implementation of the PARTAN Variant of the Linear approximation Method for the Network Equilibrium Problem. *Networks*, 17, 319-339.

Florian, M. and Nguyen, S. (1976). An application and validation of Equilibrium trip assignment models. *Transportation Science*, 10(4), 374-390.

Frank, M. and Wolfe, P. (1956). An Algorithm for Quadratic Programming. *Naval Researchs Logistic Quarterly*, 3, 95-110.

Fréchet, M.M. (1906). Sur quelques points du calcul fonctionnel. *Rendiconti del Circolo Matematico di Palermo*, 22(1), 1-72.

Frederix, R., Viti, F. and TampÃ¨re, C.M.J. (2013). Dynamic Origin-Destination Estimation in Congested Networks: Theoretical Findings and Implications in Practice. *Transp. A Transp. Sci.*, 9(6), 494-513. https://doi.org/10.1080/18128602.2011.619587.

Friesz, T.L., Bernstein, D., Smith, T.E., Tobin, R.L. and Wie, B.W. (1993). A Variational Inequality Formulation of the Dynamic Network User Equilibrium Problem. *Operations Research*, 41(1), 179-191.

Fritzsche, H.T. (1994). A model for traffic simulation. *Transp. Engin. Contr.*, 5, 317-321.

García-Albertos, P., Picornell, M., Salas-Olmedo, M. and Gutiérrez, J. (2018). Exploring the potential of mobile phone records and online route planners for dynamic accessibility analysis. *Transportation Research Part A*.

Gazis, D.C., Herman, R.L. and Rothery, R. W. (1961). Nonlinear Follow-the-Leader Models of Traffic Flow. *Operations Research*, 9(4), 545-567.

Gazis, D.C., Herman, R. and Potts, R.B. (1959). Car-Following Theory of Steady- State Traffic Flow. *Opns. Res.*, 7, 499-505.

Gentile, G. (2010). *The General Link Transmission Model for Dynamic Network Loading and a comparison with the DUE algorithm*. In *New Developments In Transport Planning: Advances In Dynamic Traffic Assignment*, edited by Immers, L.G.H., Tampere C.M.J. and F. Viti. MA, USA: Transport Economics, Management and Policy Series, Edward Elgar Publishing.

Gentile, G. (2015). Using the General Link Transmission Model in a Dynamic Traffic Assignment to simulate congestion on urban networks. *Transportation Research Procedia,* 5, 66-81.

Gerlough, D.L. and Huber, M.J. (1975). Traffic Flow Theory: A Monograph. *TRB Special Report*, 165.

González, M.C., Hidalgo, C.A. and Barabási, A.L. (2008). Understanding individual human mobility patterns. *Nature*, 453(7196), 779-782.

Guerrero-Ibáñez, J., Zeadally, S. and Contreras-Castillo, J. (2018). ISensor Technologies for Intelligent Transportation Systems. *Sensors 2018*, 18, 1212. doi:10.3390/s18041212

Gundlegård, D., Rydergren, C., Breyer, N. and Rajna, B. (2016). Travel demand estimation and network assignment based on cellular network data. *Computer Communications*, 95, 29-42. http://dx.doi.org/10.1016/j.comcom.2016.04.015

Gundlegård, D., Rydergren, C., Barcelo, J., Dokoohaki, N., Görnerup, O. and Hess, A. *Travel Demand Analysis with Differentially Private Releases*. Submitted for D4D Challenge Senegal 2014 (Netmob 2015, MIT, Boston).

Gipps, P.G. (1981). A behavioral car-following model for computer simulation. *Transp. Res. B*, 15, 105-111.

Hall, M. D., Van Vliet, D. and Willumsen, L.(1980). SATURN—a simulation-assignment model for the evaluation of traffic management schemes. *Traffic Engineering and Control*, 21(4), 168-176.

Hariharan, R. and Toyama, K. (2004). Project Lachesis: Parsing and Modeling Location Histories. In M.J. Egenhofer, C. Freksa, and H.J. Miller (Eds.): *GIScience 2004*, LNCS 3234, 106-124, 2004. Springer Verlag.

Hato, E (2010). Development of behavioral context addressable loggers in the shell for travel-activity analysis. *Transportation Research Part C*, 1855-67.

Hearn, D.W., Lawphonpanich, S. and Ventura, J.A. (1987). Restricted Simplicial Decomposition: Computation and Extensions. *Mathematical Programming Study*, 31, 99-118.

Heavens, N.G., Ward, D.S. and Natalie, M.M. (2013).Studying and Projecting Climate Change with Earth System Models. *Nature Education Knowledge*, 4(5), 4.

Herman, R., Montroll, E.W. and Potts, R.B. (1959). Traffic Dynamics: Analysis of Stability in Car Following. *Opns. Res.*, 7, 86-106.

Horni, A., Nagel, K. and Axhausen, K.W. (eds.) (2016). *The Multi-Agent Transport Simulation MATSim. London: Ubiquity Press*. DOI: http://dx.doi.org/10.5334/baw.

Iqbal Md. S., Choudhury, C.F., Wang, P. and Gonzalez, M.C. (2014). Development of Origin-Destination Matrices Using Mobile Phone Call Data. *Transportation Research Part C*, 40, 63-74.

ITF (2015). Big Data and Transport: Understanding and Assessing Options. *International Transport Forum*. OECD.

Itoh, S. and Hato, E. (2013). Combined estimation of activity generation models incorporating unobserved small trips using probe person data. *Journal of the Eastern Asia Society for Transportation Studies*, 10, 525-537.

Janmyr, J. and Wadell, D. (2018). *Analysis of Vehicle Route Choice During Incidents* (MSc thesis). University of Linköping, Department of Science and Technology, LiU-ITN-TEK-A–18/020-SE.

Jayakrisham, R., Mahmassani, H.S. and Yu, T.Y. (1994). An Evaluation Tool for Advanced Traffic Information and Management Systems in Urban Networks. *Transportation Research Part C: Emerging Technologies*, 2C(3), 129-147.

Jiang, S., Yanga, Y., Gupta, S., Veneziano, D., Athavale, S. and González, M. C. (2016). Supporting Information. *PNAS 2016*, vol. XXX, no. XX , 1-11.

Kalman, R.E. (1960). A new approach to linear filtering and prediction problem. *Journal of Basic Engineering*, 82(1), 35-45.

Kesting, A., Treiber, M. and Helbing, D. (2010). Enhanced Intelligent Diver Model to Access the Impact of Driving Strategies on Traffic capacity Simulations. *Philosophical Transactions of the Royal Society A*, 368, 4585-4605.

Kim, I-S. and McLean, W. (2013). Computing the Hausdorff distance between two sets of parametric curves. *Communications of the Korean Mathematical Society*. Oct 31; 28(4), 833-50. https://doi.org/10.4134/CKMS.2013.28.4.833

Kotsialos, A. and Papageorgiou, M. (2001). The Importance of Traffic Flow Modeling for Motorway Traffic Control. *Networks and Spatial Economics*, 1, 179-203.

Krauss, S., Wagner, P. and Gawron, C. (1997). Metastable States in a Microscopic Model of Traffic Flow. *Physical Review E*, 55(5), 55-97.

Krishnakumari, P., Van Lint, H., Djukic, T. and Cats, O. (2019). A data driven method for OD matrix estimation. *Transportation Research C*, doi: 10.1016/j.trc.2019.05.014.

Kubicka, M., Cela, A., Mounier, H. and. Niculescu, S.I. (2018). *Comparative Study and Application-Oriented Classification of Vehicular Map-Matching Methods*. doi: 10.1109/MITS.2018.2806630

Lawphongpanich, S. and Hearn, D.W. (1984). Simplicial Decomposition of the Asymmetric Traffic Assignment Problem. *Transportation Research 18B*, 123-133.

LeBlanc, L.J., Morlok, E.K. and Pierskalla, W.P. (1975). An Efficient Approach for Solving the Road Network Equilibrium Traffic Assignment Problem. *Transportation Research*, 5, 309-318.

Levinson, D.M. and Kumar, A. (1994). The rational locator: why travel times have remained stable. *Journal of the American Planning Association*, 60(3), 319-332.

Lighthill, M. and Whitham, G. (1955). On Kinematic Waves II: A Traffic Flow Theory on Long Crowded Roads. *Proceedings of the Royal Society of London Series A*, 229, 317-345.

Lopez, C., Krishnakumari, P., Leclercq, L., Chiabaut, N. and Van Lint, H. (2017a). Spatiotemporal Partitioning of Transportation Network Using Travel Time Data. *Transportation Research Record: Journal of the TRB*, 2623, 98-107.

Lopez, C., Leclercq, L., Krishnakumari, P., Chiabaut, N. and Van Lint, H. (2017b). Revealing the day-to-day regularity of urban congestion patterns with 3D speed maps. *Scientific Reports*, 2(7), 14029.

Lu, X-Y. and Skabardonis, A. (2007). Freeway Traffic Shockwave Analysis: Exploring the NGSIM Trajectory Data. *TRB 86th Annual Meeting Compendium of Papers CD-ROM*, Paper #07-3016.

Mahmassani, H. (2001). Dynamic Network Traffic Assignment and Simulation Methodology for Advanced System Management Applications. *Network and Spatial Economics*, 1, 267-292.

Mahmassani, H.S., Hu, T.Y., Peeta, S. and Ziliaskopoulos, A. (1994). Development and Testing of Dynamic traffic assignment and Simulation procedures for ATIS/ATMS

Applications. Technical Report DTFH61-90-R00074-FG, Center for Transportation research, The University of Texas at Austin.

Mahut, M. and Florian, M. (2010). *Traffic Simulation with Dynameq.* In *Fundamentals of Traffic Simulation*, edited by Barceló, J. Switzerland: Springer, ISBN 978-1-4419-6142-6.

Mahut, M. (1999). *Behavioural Car Following Models. Report CRT-99-31*. Centre for Research on Transportation. University of Montreal. Montreal, Canada.

Mahut, M. (2001). *Discrete flow model for dynamic network loading*. Ph.D. Thesis, Département d'Informatique et de Recherhe Opérationelle, Université de Montréal. Published by the Center for Research on Transportation (CRT), University of Montreal.

McNally, M.G. (2000). *The Four Step Model*. UCI-ITS-AS-WP-00-5. Institute of Transportation Studies and Department of Civil & Environmental Engineering University of California, Irvine. http://www.its.uci.edu

Michalopoulos, P.G., Yi, P. and Lyrintzis, A.S. (1993). Continuum modelling of traffic dynamics for congested freeways. *Transportation Research Part B: Methodological*, 27(4), 315-332.

Michalopoulos, P.G. (1984). Dynamic Freeway Simulation Program for Personal Computers. In *Transportation Research Records 971, National Research Council*, Washington, D.C., 68-79.

Michalopoulos, P.G., Beskos, D.E. and LinJ, K. (1984). Analysis of Interrupted Traffic Flow by Finite Difference Methods. *Transportation Research*, 18B, 409-421.

Millard-Ball, A., Hampshire, R.C. and Weinberger, R.R. (2019). Map-matching poor-quality GPS data in urban environments: the pgMapMatch package. *Transp. Plan. Technol.*, 42(6), 539-553. doi: 10.1080/03081060.2019.1622249.

Minsky, M. (1965). Matter, Mind and Models. In *Proceedings of IFIP Congress*, edited by I. F. of Information Processing Congress, 45-49.

Mitchell, R.B. and Rapkin, C. (1954). *Urban Traffic: A Function of Land Use*, Columbia University Press, New York, NY.

Mitra, A., Attanasi, A., Meschini, L. and Gentile, G. (2020). Methodology for O-D Matrix Estimation Using the Revealed Paths of Floating Car Data on Large-scale Networks. *IET Intell. Transp. Syst.*, 14(12), 1704-1711.

Montanino, M. and Punzo, V. (2015). Trajectory data reconstruction and simulation-based validation against macroscopic traffic patterns. *Transportation Research Part B*, 80, 82-106.

Montero, L., Ros-Roca, X., Herranz, R. and Barceló, J. (2019). Fusing mobile phone data with other data sources to generate input OD matrices for transport models. *Transportation Research Procedia* (www.sciencedirect.com), 37, 417-424.

MULTITUDE (2014). *Traffic Simulation: Case for Guidelines*. COST Action TU0903 MULTITUDE, European Commission Joint Research Centre Institute for Energy and Transport, http://iet.jrc.ec.europa.eu/

Newell, G.F. (1961). Nonlinear Effects in the Dynamic of Car-Following. *Operations Research*, 9(2).

Nicolas, J.P. and Zuccarello, P. (2011). Land-Use and transport Interaction Modeling: The SIMBAD Project and the Articulation between Visum and URBANSIM. *PTV Vision International Users Group Meeting*, New York, September 2011

OECD/ITF (2015). *Big data and Transport: Understanding and Assessing Options*. International Transport Forum. https://www.oecd-ilibrary.org/transport/big-data-and-%09transport_5jlwvzdb6r47-enhttps://www.oecd-ilibrary.org/transport/big-data-and-transport_5jlwvzdb6r47-en

Ortúzar, J.D. and Willumsen, L.G. (2011). *Modelling Transport*, John Wiley.

Osorio, C. (2019). Dynamic Origin-Destination Matrix Calibration for Large-Scale Network Simulators. *Transp. Res. Part C Emerg. Technol.*, 98 (April 2018), 186-206. https://doi.org/10.1016/j.trc.2018.09.023.

Papageorgiou, M. (1998). Some remarks on macroscopic traffic flow modelling. *Transportation Research Part A: Policy and Practice*, 32(5), 323-329.

Papageorgiou, M., Blosseville, J. and Hadj-Salem, H. (1990). Modelling and Real-Time Control of Traffic Flow on the Southern Part of Boulevard Peripherique in Paris. *Part I: Modelling, Transportation Research A*, 24, 345-359.

Papageorgiou, M., Blosseville, J.M. and Hadj-Salem, H. (1989). Macroscopic modelling of traffic flow on the Boulevard Périphérique in Paris. *Transportation Research Part B: Methodological*, 23(1), 29-47.

Patriksson, M. (1994). *The Traffic Assignment Problem*, VSP. (the pdf of this book is available on Patriksson's web page at http://www.cs.chalmers.se/~mipat/traffic.html

Payne, H. (1979). FREFLO: A macroscopic simulation model of freeway traffic. *Transportation Research Record*, 772, 68-75.

Payne, H.J. (1971). Models of freeway traffic and control. *Mathematical Models of Public Systems*, 1(1), 51-61.

Petrik, O., Moura, F. and Abreu e Silva, J. (2014). The Influence of the Volume-Delay Function on Uncertainty Assessment for a Four-Step Model. *Advances in Intelligent Systems and Computing*, 262, 293-306. DOI: 10.1007/978-3- 319-04630-3 22.

Pipes, L.A. (1953). An Operational Analysis of Traffic Dynamics. *Journal of Applied Physics*, 24(3).

Popper, K.R. (1972). *Conjectures and Refutations. The Growth of Scientific Knowledge*. Routledge & Kegan, London.

PTV AG (2020). PTV Visum 2020 - User's Manual. PTV Group, Karlsruhe, Germany.

Punzo, V., Borzacchiello, M.T. and Ciuffo, B. (2011). On the assessment of vehicle trajectory data accuracy and application to the Next Generation SIMulation (NGSIM) program data. *Transportation Research Part C*, 19, 1243-1262.

Ran, B. and Boyce, D. (1996). *Modeling Dynamic Transportation Networks*, Springer-Verlag.

Ratti, C., Frenchman, D., Pulselli, R.M. and Williams, S. (2006). Mobile landscapes: Using location data from cell phones for urban analysis. *Environment and planning B: Planning and design*, 33(5), 727-748.

Richards, P. (1956). Shock Waves on the Highway. *Operations Research*, 4, 42-51.

Rosen, B. (1960). The gradient projection method for nonlinear programming I. Linear Constraints. *J. Soc. Indust. Appl. Math.*, 9, 181-217.

Ros-Roca, X., Montero, L., Barceló, J., Nökel, K. and Gentile, G. (2022). A practical approach to assignment-free Dynamic Origin–Destination Matrix Estimation problem. *Transportation Research Part C*, 134, https://doi.org/10.1016/j.trc.2021.103477

Ros-Roca, X., Montero, L. and Barceló, J. (2020). Investigating the quality of Spiess-like and SPSA approaches for dynamic od matrix estimation. *Transportmetrica*. https://doi.org/10.1080/23249935.2020.1722282.

Rothery, R.W. (2001). *Car-Following Models. Chapter 4 in TRB (2001)*. Revised Monograph on Traffic Flow Theory.

Rouphail, N.M. and Sacks, J. (2003). *Thoughts on Traffic Models Calibration and Validation*, paper presented at the Workshop on Traffic Modeling, Sitges, Spain.

Sakaia, T., Alho, A.R., Bhavathrathan, B.K., Dalla Chiara, G., Gopalakrishnan, R., Jing, P., Hyodo, T., Cheah, L. and Ben-Akiva, M. (2020). SimMobility Freight: An agent-based urban freight simulator for evaluating logistics solutions. *Transportation Research Part E*, 141. https://doi.org/10.1016/j.tre.2020.102017

Savitzky, A. and Golay, M.J.E. (1964). Smoothing and Differentiation of Data by Simplified Least Squares Procedures. *Analytical Chemistry*, 36(8), 1627-39.

Schafer, A. (2000). Regularities in travel demand: an international perspectiv. *Journal of transportation and statistics*, 3(3), 1-31.

Sheffi, Y. (1985). *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*. Prentice Hall. (Available from Prof. Sheffi's web site: http://mit.edu/sheffi/www/urbanTransportation)

Smith, M.J. (1993). A new dynamic traffic model and the existence and calculation of dynamic user equilibria on congested capacity-constrained road networks. *Transportation Research Part B*, 27, 49-63.

Smith, M.J. (1979). Existence, Uniqueness and Stability of Traffic Equilibria. *Transportation Research B*, 1B, 295-304.

Spall, J.C. (1992). Multivariate Stochastic Approximation Using a Simultaneous Perturbation Gradient Approximation. *IEEE Trans. Automat. Contr.*, 37(3), 332-341. https://doi.org/10.1109/9.119632.

Spiess, H. (1990). Conical volume delay functions. *Transportation Science*, 24(2), 153-158.

Stock, W.A., Blankenhorn, R.C. and May, D.C. (1973). *The FREQ3 Freeway Model*, ITTE Report 73-2, University of California Berkeley

Talebpour, A. and Mahmassani, H.S. (2016). Influence of connected and autonomous vehicles on traffic flow stability and throughput. *Transportation Research Part C*, 1, 143-163.

Talebpour, A., Mahmassani, H.S. and Bustamante, F.E. (2016). Modeling Driver Behavior in a Connected Environment Integrated Microscopic Simulation of Traffic and Mobile Wireless Telecommunication Systems. *Transportation Research Record: Journal of the Transportation Research Board*, No. 2560, Transportation Research Board, Washington, D.C., 2016, 75-86. DOI: 10.3141/2560-09

Toledo, T. and Kolechkina, T. (2013). Estimation of Dynamic Origin-Destination Matrices Using Linear Assignment Matrix Approximations. *IEEE Trans. Intell. Transp. Syst.*, 14(2), 618-626. https://doi.org/10.1109/TITS.2012.2226211.

Toole, J.L., Colak, S., Sturt, B., Alexander, L.P., Evsukoff, A. and González M.C. (2015). The path most traveled: Travel demand estimation using big data resources. *Transportation Research Part C*, 58, 162-177.

TRB (2001). Revised Monograph on *Traffic Flow Theory*. https://www.fhwa.dot.gov/publications/research/operations/tft/

Treiber, M. and Kesting, A. (2013). *Traffic Flow Dynamics. Data, Models and Simulation*. Springer ISBN 978-3-642- 32459-8.

van Wee, B. (2015). Viewpoint: Toward a new generation of land use transport interaction models. *The Journal of Transport and Land Use*, 8(3), 1-10. http://jtlu.org

Viterbi, A.J. (1967). Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. *IEEE Transactions on Information Theory*, 13(2), 260-269.

Waddell, P. (2015). Integrated transport and land use modeling for sustainable cities. Michel Bierlaire, Andrè De Palma, Ricardo Hurtubia, Paul Waddell. EPFL Press.

Waddell, P., Boeing, G., Gardner, M. and Porter, E. (2018). An Integrated Pipeline Architecture for Modeling Urban Land Use. *Travel Demand, and Traffic Assignment*. arXiv:1802.09335v1 [cs.CY].

Waddell, P. (2011). Integrated Land Use and Transportation Planning and Modelling: Addressing Challenges in Research and Practice. *Transport Reviews: A Transnational Transdisciplinary Journal*, 31(2), 209-229, DOI: 10.1080/01441647.2010.525671

Wagner, P. (2016). Traffic control and traffic management in a transportation system with autonomous vehicles. Ch. 15 in M. Maurer et al. (eds). *Autonomous Driving*. DOI 10.10007/978-3-662-48847-8-15.

Ward, J.A. (2009). *Heterogeneity, Lane-Changing and Instability in Traffic: A Mathematical Approach*. Department of Engineering Mathematics. University of Bristol, Bristol, United Kingdom.

Ward, J.A. and Wilson, R.E. (2011). Criteria for convective versus absolute string instability in car-following models. *Proc. R. Soc. A*, 467, 2185-2208. doi:10.1098/rspa.2010.0437

Wardrop, J.G. (1952). Some Theoretical Aspects of Road Traffic Research. *Proc. Inst. Civil Engineers, Part II*, 325-378.

Wegener, M. and Fürst, F. (1999). Land-Use Transport Interaction: State of the Art. Deliverable 2a of the project TRANSLAND (Integration of Transport and Land Use Planning) of the 4th RTD Framework Programme of the European Commission. Institut für Raumplanung. Fakultä Raumplanung, Universität Dortmund.

Wiedemann, R. (1974). Simulation des Verkehrsflusses. Schriftenreihe des Instituts für Verkehrwessen, Heft 8, Universität (TH) Karlsruhe.

Wilson, E. (2011). Mathematical Analysis of CF Models, NEARCTIS Course (Delft 2011).

Wu, J.H. (1991). *A Study of Monotone Variational Inequalities and their Application to Nertwork Equilibrium Problems*, Ph. D. Thesis, Centre de Recherche sur les Transports, Université de Montréal, Publication #801.

Wu, J.H., Chen, Y. and Florian, M. (1998a). The Continuous Dynamic Network Loading Problem: A Mathematical Formulation and Solution Method. *Trans. Res.-B*, 32(3), 173-187.

Wu, J.H., Florian, M., Xu, Y.W. and Rubio-Ardanaz, J.M. (1998b). A projection algorithm for the dynamic network equilibrium problem. *Traffic and Transportation Studies, Proceedings of the ICTTS'98*, 379-390, Ed. By Zhaoxia Yang, Kelvin C.P. Wang and Baohua Mao, ASCE.

Wu, C., Kreidiehy, A., Parvate, K., Vinitskyz, E. and Bayen, A. (2017). Flow: Architecture and Benchmarking for Reinforcement Learning in Traffic Control. arXiv:1710.05465v1 [cs.AI]

Yeo, Y. and Skabardonis, A. (2007). TO 9: Oversaturated Freeway Flow Algorithm, Final Report, University of California Berkeley, NGSIM Program.

Zhang, Y., Qin, X., Dong, S. and Ran, B. (2010). Daily O-D Matrix Estimation Using Cellular Probe Data, Paper 10- 2472, presented at the 89^{th} TRB Annual Meeting, included in the Compendium of Papers.

Zhu, Y., Ferreira, J., Diao, M. and Zegras, P.C. (2018). An integrated microsimulation approach to land-use and mobility modeling. *The Journal of Transport and Land Use*, 11(1), 633-659. http://jtlu.org